PHYS/PHIL329 Lecture 25: Possible solutions to the probability problem

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Announcements

- Problem Set 4: due May 5, 11.59pm.
- Problem Set 5: due May 12, 11.59pm.
- Essay 5: posted on April 30, due May 13, 11.59pm.
- Final exam has been posted, due May 22, 11.59pm.

Recap

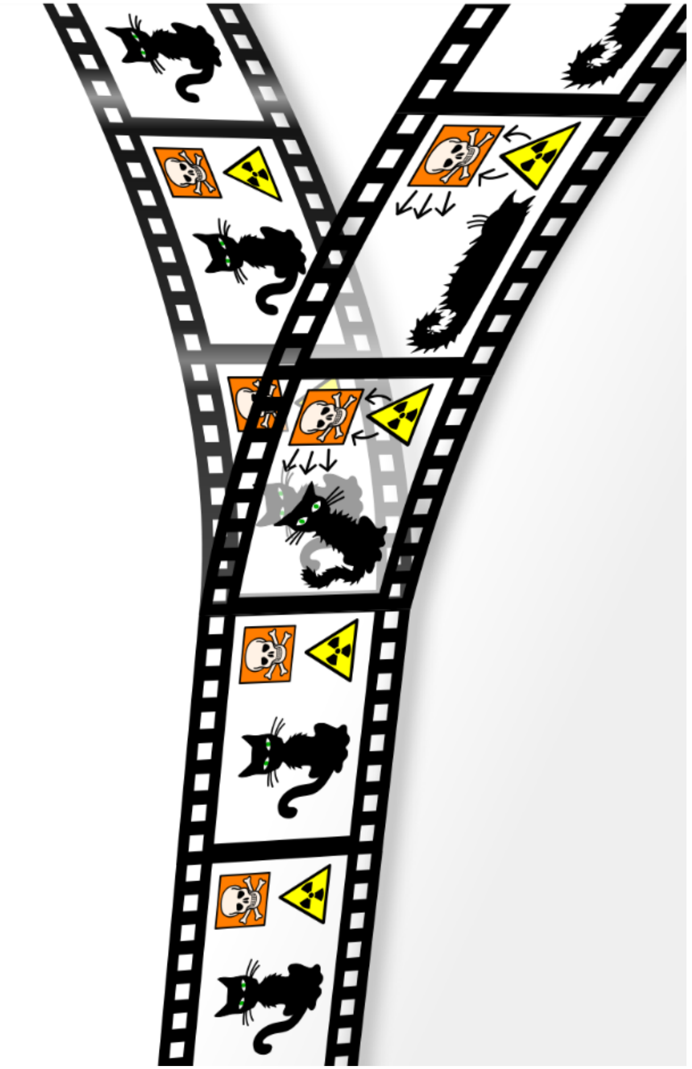
- According to many worlds:
  - A system in *a macroscopic superposition* should be interpreted as *many macroscopic systems*, each macroscopic system is in its own world.

$$\frac{1}{\sqrt{2}}|'black'\rangle_{you}|"black"\rangle_m|black\rangle_e + \frac{1}{\sqrt{2}}|'white'\rangle_{you}|"white"\rangle_m|white\rangle_e$$

- Primary motivations:
  - *Theoretical simplicity*: many worlds formalism only requires state vector and the linear dynamics.
  - *Locality*: by eliminating non-local processes from the theory (like collapse), many worlds is easier to reconcile with relativity theory.
- Worlds as emergent decoherence-defined entities:
  - Worlds are not to be strictly identified with the components of the state vector in one special basis, nor are they to be added to the formalism.
  - Worlds are patterns that inevitably emerge in state vectors that are written down in bases in which environmental decoherence occurs.
    - https://www.youtube.com/watch?v=GIOwJWJWPUs

Today's Lecture

- The three probability problems.
  - Incoherence problem.
  - Quantitative problem.
  - Confirmation problem.
- Possible solution 1:
  - The "it's a problem for everyone so it's a problem for no one" strategy.
  - David Papineau's arguments.
- Possible solution 2:
  - The decision-theoretic (Oxford) strategy.
  - The Deutsch-Wallace proof.
- Possible solution 3:
  - The self-location uncertainty strategy.
  - The McQueen-Vaidman proof.



Three probability problems

- The incoherence problem
  - No sense can be made of non-trivial probabilities (other than 0 and 1), because an experimenter (who believes many worlds) knows for certain that all possible outcomes occur with probability 1.

- The quantitative problem
  - If the many worlds interpretation is true then the Born Rule is false: the only probability rule that many worlds supports is the Equal Probability Rule.

- The confirmation problem
  - The many worlds interpretation is unfalsifiable, or at least, cannot be falsified by the kinds of low-probability experimental outcomes that would falsify quantum mechanics. After all, every possible string of experimental results is actually observed in some world and only in a vast minority of worlds do experimenters observe Born Rule statistics.

An example

- Compare experiment 1:
  - $|\psi\rangle_{t_2} = |\text{"}ready\text{"}\rangle_{you}\left(\frac{1}{\sqrt{2}}|\text{"}black\text{"}\rangle_m|black\rangle_e + \frac{1}{\sqrt{2}}|\text{"}white\text{"}\rangle_m|white\rangle_e\right)$
  - $|\psi\rangle_{t_3} = \frac{1}{\sqrt{2}}|'black'\rangle_{you}|\text{"}black\text{"}\rangle_m|black\rangle_e + \frac{1}{\sqrt{2}}|'white'\rangle_{you}|\text{"}white\text{"}\rangle_m|white\rangle_e$
- With experiment 2:
  - $|\varphi\rangle_{t_2} = |\text{"}ready\text{"}\rangle_{you}\left(\frac{1}{2}|\text{"}black\text{"}\rangle_m|black\rangle_e + \sqrt{3/4}\,|\text{"}white\text{"}\rangle_m|white\rangle_e\right)$
  - $|\varphi\rangle_{t_3} = \frac{1}{2}|'black'\rangle_{you}|\text{"}black\text{"}\rangle_m|black\rangle_e + \sqrt{3/4}\,|'white'\rangle_{you}|\text{"}white\text{"}\rangle_m|white\rangle_e$
- Incoherence problem:
  - In both experiments, you know at t_2 that both outcomes will occur with certainty, so non-trivial probability assignments won't mean anything.
- Quantitative problem:
  - Even if you could assign meaningful probabilities, they won't correspond to the Born rule. The most plausible rule here is the equal probability rule: since both experiments end with two equally likely worlds.
- Confirmation problem:
  - Imagine you plan to run experiment 2 many times. Why should you be surprised if you *don't* get roughly 25/75 results, but instead get the kind of 50/50 results that we normally expect from running experiment 1 many times? How are these two experimental set-ups distinguishable?

Possible solution 1 (Papineau)

"[those] who criticize Everettians about probability are a classic case of a pot calling a kettle black"

- Many worlds:
  - 1. After exp. 2, *both* (black & white) outcomes occur, so it is not explained why black has probability 0.25.
  - 2. After multiple runs of exp. 2, there *are multiple branches* of outcomes, most don't have 25% black electrons, so it is not explained why black has probability 0.25.
  - 3. After multiple runs of exp. 2, an experimenter can try to estimate the true probability, based on the outcomes on their branch. But who's to say that *their* branch captures the true probability, and not some other branch?
  - 4. After a hypothetical *infinite sequence* of runs of exp. 2, many worlds cannot identify the probability of black with *the* relative frequency of black outcomes, since *all possible* frequencies obtain.
- Orthodox (one world) view:
  - 1. After exp. 2, *one* outcome occurs, this also doesn't explain why black has probability 0.25.
  - 2. After multiple runs of exp. 2, there is *one branch* of outcomes, the ratio of black/white is typically not exactly 0.25/0.75, so this also doesn't explain why black has probability 0.25.
  - 3. After multiple runs of exp. 2, an experimenter can try to estimate the true probability, based on the outcomes. But who's to say that the actual outcomes capture the true probability, perhaps the experimenter saw an improbable string of outcomes?
  - 4. After a hypothetical *infinite sequence* of runs of exp. 2, the textbook view also cannot identify the probability of black with the relative frequency of black outcomes, since they too can come apart.

Probability and the Principal principle

- Here is a definition of probability from a well-known engineering textbook:
  - "The probability of an outcome is the proportion of times the outcome would occur in a long run of repeated experiments."
    - Johnson, R.A. (1994). Miller & Freund's probability & statistics for engineers (5th ed.). p57.
- Papineau's (and the Oxford Everettian's) objection:
  - Whether the "long run" is infinite, or just really long, the fallacy is the same:
    - Trying to give *a definitive answer* as to how *a probabilistic system* will behave fundamentally misunderstands probability.
  - The *real* connection between probability and frequency:
    - [Insert textbook definition]… *probably*.
    - This *circular* definition is obviously open to Everettians.
- A more fundamental definition of probability:
  - *Principal principle*: Probability is what constrains rational credence (degree of belief).
    - We can *measure* your credences by observing your *betting behaviour*.
    - The Oxford Everettians can *prove the Born rule*, if they can prove that a rational agent who believes many worlds and knows the full quantum state *ought to bet in accord with the Born rule*.
    - This will solve the quantitative problem.

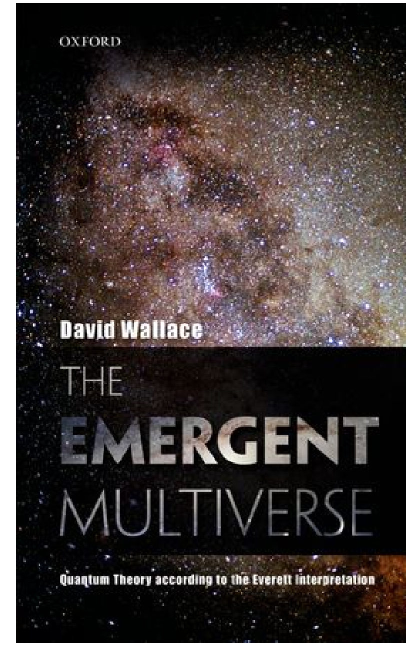
Possible solution 2: the Deutsch-Wallace proof

- Step 1: Which quantum game would you prefer to play?
  - G1: $\frac{1}{\sqrt{2}}|\$5\rangle_m|black\rangle_e + \frac{1}{\sqrt{2}}|\$0\rangle_m|white\rangle_e$
  - G2: $\frac{1}{\sqrt{2}}|\$0\rangle_m|black\rangle_e + \frac{1}{\sqrt{2}}|\$5\rangle_m|white\rangle_e$
    - Claim: the two games are equivalent (apart from irrelevant labelling), so a rational agent will be *indifferent* to playing G1 versus G2.
    - But that is to *act as if* the two branches (in either game) are *equally probable*.

- Step 2: Which quantum game would you prefer to play?
  - G3: $\frac{1}{2}|\$0\rangle_m|black\rangle_e + \sqrt{3/4}\,|\$5\rangle_m|white\rangle_e$
  - G4: $\frac{1}{2}|\$0\rangle_m|X_1\rangle_e + \frac{1}{2}|\$5\rangle_m|X_2\rangle_e + \frac{1}{2}|\$5\rangle_m|X_3\rangle_e + \frac{1}{2}|\$5\rangle_m|X_4\rangle_e$
    - Claim: the two games are equivalent (apart from irrelevant labelling and *apart from irrelevant branching of the G4 reward branch into the three X_2 X_3 X_4 branches*), so a rational agent will be indifferent to playing G3 versus G4.
    - But that is to *act as if* the G3 reward branch is *3 times more probable* than the G3 non-reward branch.

The "branching indifference" axiom

- Branching indifference:
  - An agent doesn't care about branching per se: if a certain operation leaves her future selves in N different macrostates but doesn't change any of their rewards, she is indifferent as to whether or not the operation is performed.
    - Branching the white branch into three branches need not affect how much money the machine pays, so you should be indifferent to such branching.



- Why believe it?
  - The pragmatic defense:
    - "A preference which is not indifferent to branching per se would in practice be *impossible to act on*: branching is uncontrollable and ever-present" (Wallace, 2012: 170).
  - The non-existence defense:
    - A preference which is not indifferent to branching per se is *meaningless*: it would require there to be a determinate branch count.

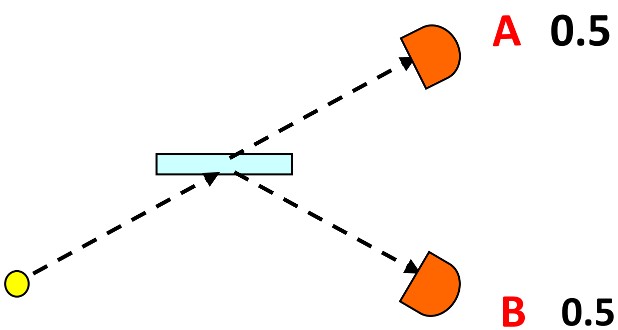
Possible solution 3: self-location uncertainty

- Lev Vaidman (1998) tried to solve the incoherence problem by showing how *self-location probabilities* make sense in many worlds.

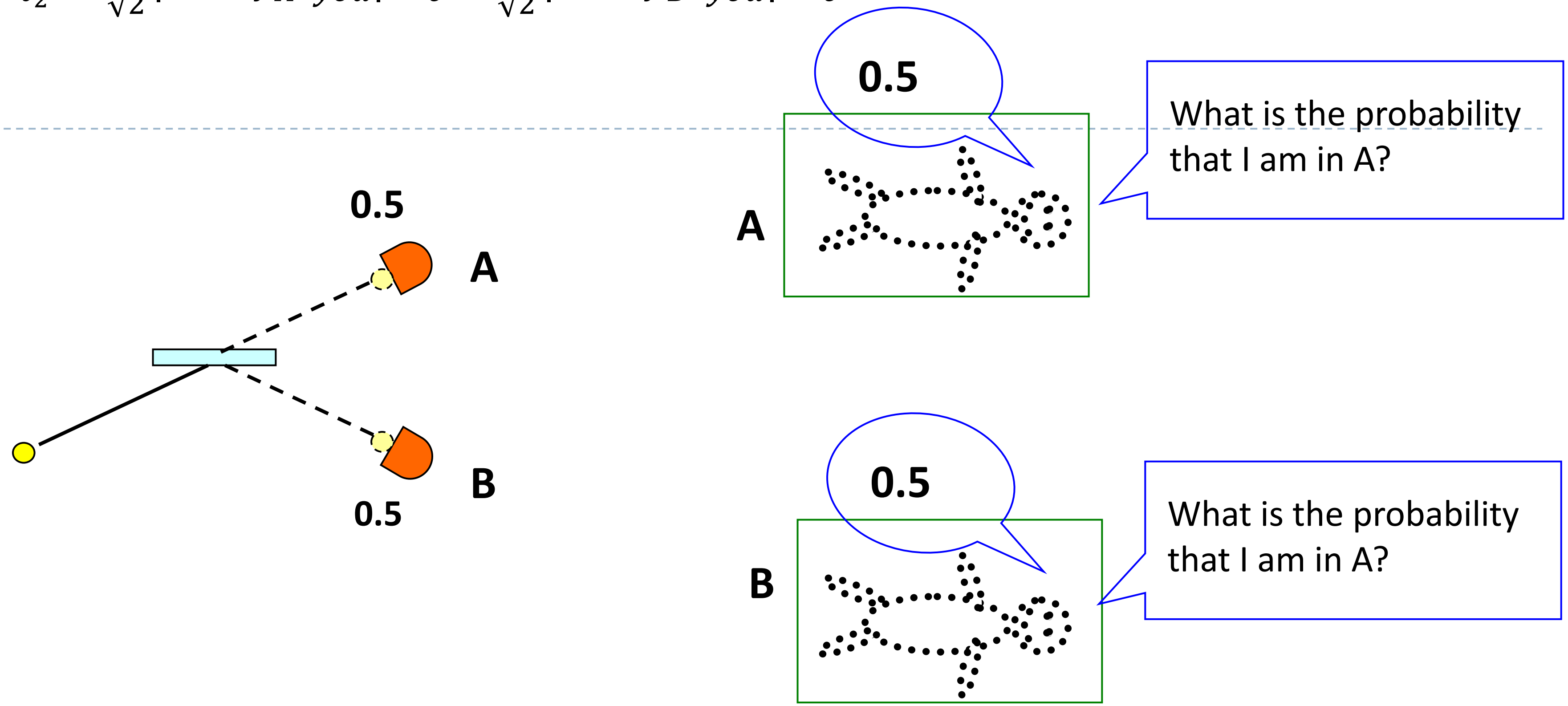
- Sleeping pill thought experiment:
  - You are blindfolded. You run the experiment.
  - If you measure A, then you are put to sleep and placed in room A.
  - If you measure B, then you are put to sleep and placed in room B.
  - Rooms A and B are internally identical.
  - When you wake, you can meaningfully ask about the probability that you are *located* in the A(B) room/world.

$$|\psi\rangle_{t_1} = |ready\rangle_{you}\left(\frac{1}{\sqrt{2}}|A\rangle_e + \frac{1}{\sqrt{2}}|B\rangle_e\right)$$

$$|\psi\rangle_{t_2} = \frac{1}{\sqrt{2}}|ready_A\rangle_{you}|A\rangle_e + \frac{1}{\sqrt{2}}|ready_B\rangle_{you}|B\rangle_e$$

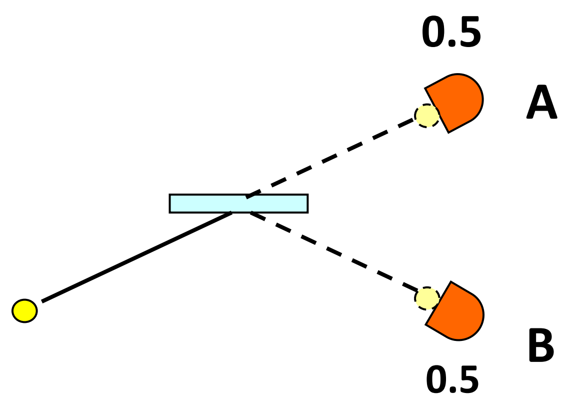


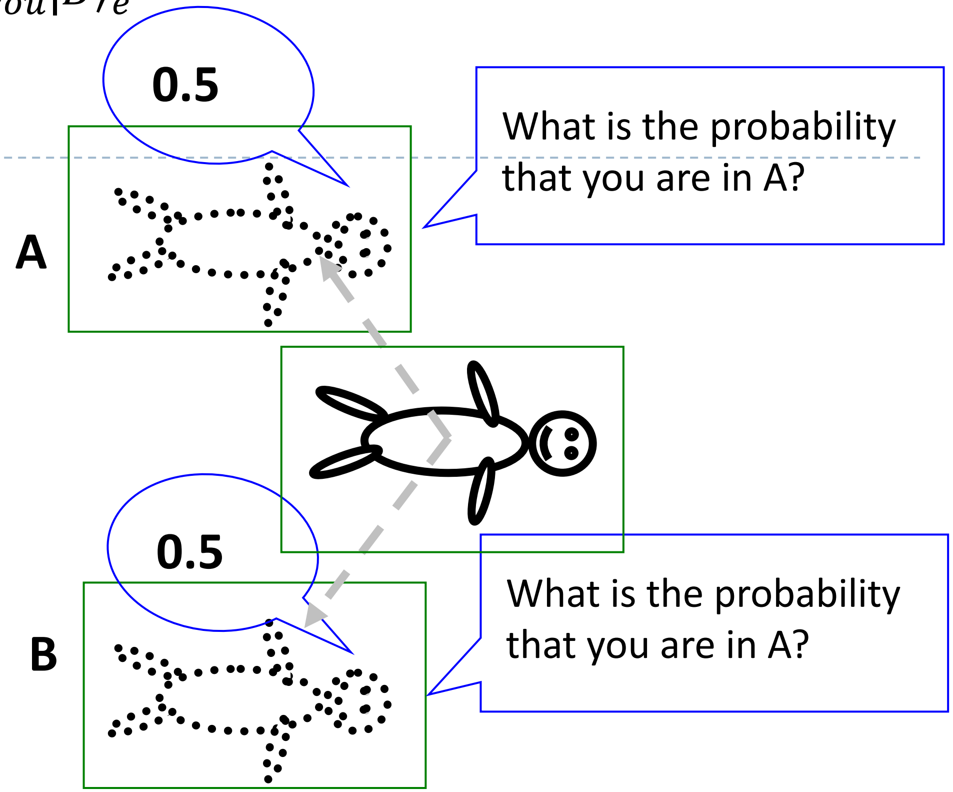
$$|\psi\rangle_{t_2} = \frac{1}{\sqrt{2}}|ready_A\rangle_{you}|A\rangle_e + \frac{1}{\sqrt{2}}|ready_B\rangle_{you}|B\rangle_e$$



- The two descendants know $|\psi\rangle_{t_2}$ so they can infer the correct (Born rule) self-location probabilities.
- Three challenges now arise:
  - 1. These are the descendants' *backwards-looking* probability assignments. What about the ancestor's *forwards-looking* probability assignments?
  - 2. Experimenters are typically not blindfolded and drugged. How can this contrived set-up generalize?
  - 3. The 0.5 probability assignments are consistent with the Born rule. But they are also consistent with the Equal probability rule. What solves the quantitative problem?

$$|\psi\rangle_{t_2} = \frac{1}{\sqrt{2}}|ready_A\rangle_{you}|A\rangle_e + \frac{1}{\sqrt{2}}|ready_B\rangle_{you}|B\rangle_e$$





- Challenge 1:
  - These are the descendants' *backwards-looking* probability assignments. What about the ancestor's *forwards-looking* probability assignments?
- Response:
  - Since all descendants give the same answer, we can relate that answer to the pre-experiment ancestor.
  - If you were choosing between quantum games, your descendants would want you to maximize the rewards on the most probable branches.
  - Since your descendants reap the rewards, you bet in accord with the Born rule.

Generalizing the sleeping pill experiment

- Challenge 2:
  - Experimenters are typically not blindfolded and drugged. How can this contrived set-up generalize?
- First distinguish:
  - Absent self-location uncertainty

$$|\psi\rangle_{absent} = \frac{1}{\sqrt{3}} |\uparrow\rangle_p |\square, \uparrow\rangle_n |see : \square\rangle_y + \sqrt{\frac{2}{3}} |\downarrow\rangle_p |\square, \downarrow\rangle_n |see : \square\rangle_y$$

  - Tainted self-location uncertainty:

$$|\psi\rangle_{tainted} = \frac{1}{\sqrt{3}} |\uparrow\rangle_p |\square, \uparrow\rangle_n |see : \square\rangle_y + \sqrt{\frac{2}{3}} |\downarrow\rangle_p \left|\Diamond, \downarrow\right\rangle_n \left|see : \Diamond\right\rangle_y$$

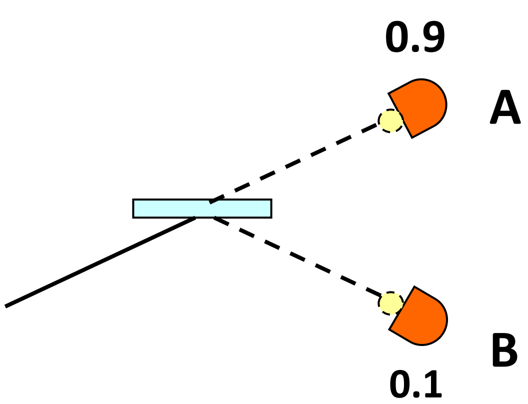
  - Clean self-location uncertainty:

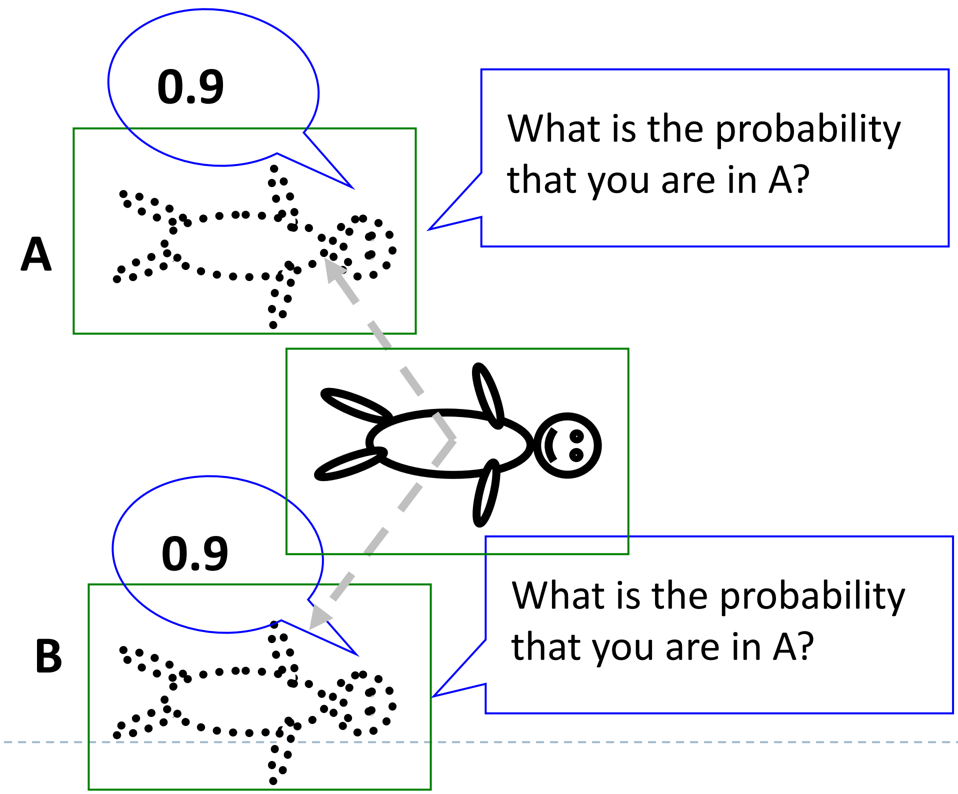
$$|\psi\rangle_{clean} = \frac{1}{\sqrt{3}} |\uparrow\rangle_p |\square, \uparrow\rangle_n |see : \square\rangle_y + \sqrt{\frac{2}{3}} |\downarrow\rangle_p \left|\Diamond, \downarrow\right\rangle_n \left|see: \Diamond\right\rangle_y$$

- Response:
  - *Clean* situations are indeed contrived but *tainted* situations are ubiquitous.
  - Just reading about the setup of a performed experiment usually puts one in a position of *tainted* self-location uncertainty. Before reading the results section one can calculate the probability that one is in a world with a given set of results.

Solving the quantitative problem

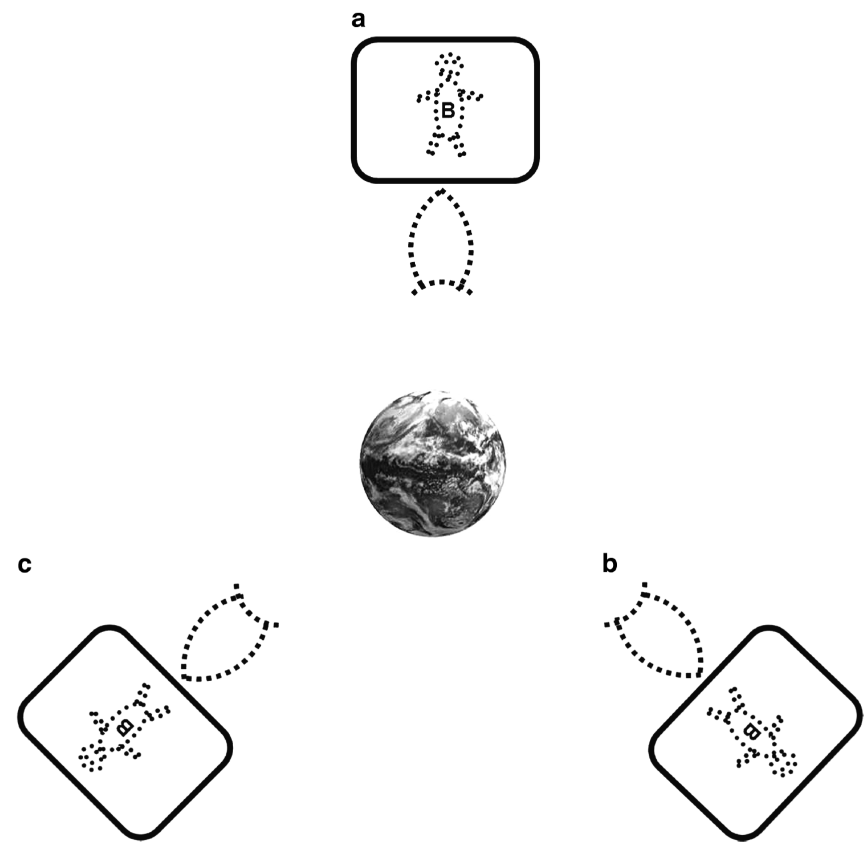
- Challenge 3:
  - The 0.5 probability assignments are consistent with the Born rule. But they are also consistent with the Equal probability rule. What solves the quantitative problem?
- Response:
  - The descendants know the quantum state $|\psi\rangle_{t_2} = \sqrt{0.9}|ready_A\rangle_{you}|A\rangle_e + \sqrt{0.1}|ready_B\rangle_{you}|B\rangle_e$
  - IF they can use $|\psi\rangle_{t_2}$ *to prove the Born rule*, then their probability assignment can guide their ancestor's bets.





Recap: proof of Born rule in collapse theories

- *Axioms:*
  - *Psi-ontic axiom*: all facts depend upon the quantum state.
  - *Probabilistic Symmetry axiom*: If there is no difference in the physical description of two places, and the same kind of measurements are performed in these places, then the probabilities of the outcomes in the two places have to be the same.
  - *No superluminal signaling axiom*: signals cannot be sent faster than light speed.

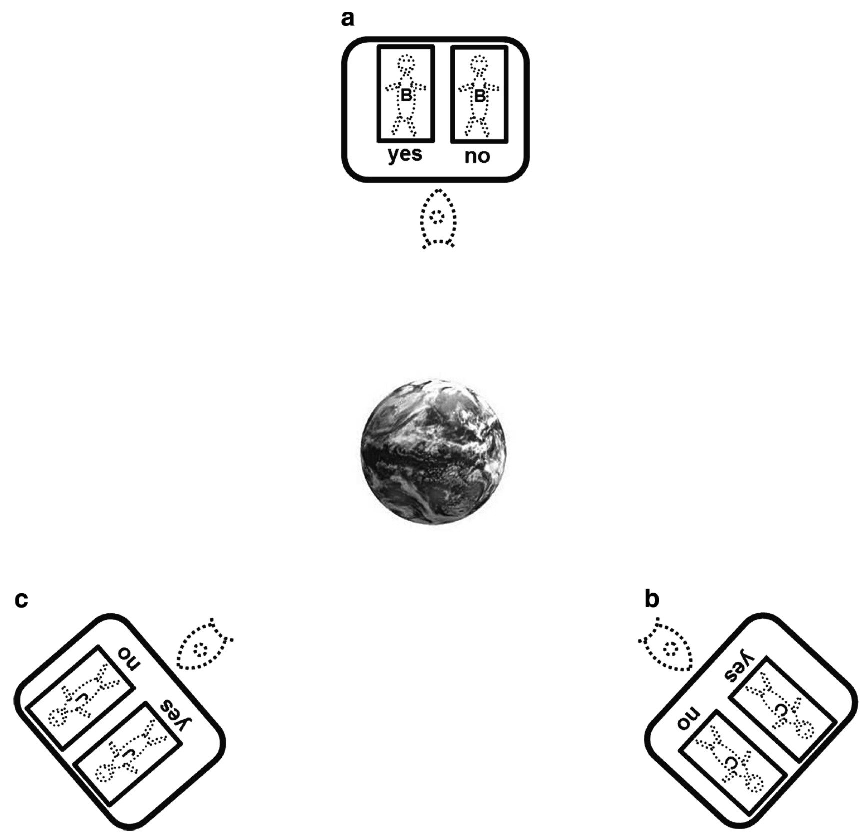


$$|\Psi\rangle = \frac{1}{\sqrt{3}}(|a\rangle + |b\rangle + |c\rangle)$$

- First two steps of proof:
  - Step 1: by symmetry, the collapse at each region must obey the Born rule.
  - Step 2: by no-signaling, combining $|b\rangle$ and $|c\rangle$ cannot affect the probabilities of outcomes in region a.

Proof of Born rule in many worlds

- ***Axioms:***
  - *Psi-ontic axiom*: all facts depend upon the quantum state.
  - *Probabilistic Symmetry axiom*: If there is no difference in the physical description of two places, and the same kind of measurements are performed in these places, then the probabilities of the outcomes in the two places have to be the same.
  - *Local supervenience*: whatever happens in a region depends only on the quantum description of that region and its immediate vicinity.
- First two steps of proof:
  - Step 1: by symmetry, the self-location probabilities for each region must obey the Born rule.
  - Step 2: by local supervenience, combining $|b\rangle$ and $|c\rangle$ cannot affect the local description, and so cannot change the self-location probabilities of region a.



$$\frac{1}{3}\,|1\rangle_{\mathrm{a}}\,|\checkmark\rangle_{\mathrm{A}}\,|\checkmark\rangle_{\mathrm{A1}}\,\langle\checkmark|_{\mathrm{A1}}\,\langle\checkmark|_{\mathrm{A}}\,\langle 1|_{\mathrm{a}}+\frac{2}{3}\,|0\rangle_{\mathrm{a}}\,|R\rangle_{\mathrm{A}}\,|R\rangle_{\mathrm{A1}}\,\langle R|_{\mathrm{A1}}\,\langle R|_{\mathrm{A}}\,\langle 0|_{\mathrm{a}}$$

The confirmation problem

- Hemmo and Pitowsky [2007]:
  - Why would our observed statistics be any more indicative of the true theory than the frequencies observed by experimenters in maverick worlds?
  - Indeed, perhaps such experimenters have concocted their own physical theory, and their own probability rule, and are exploiting symmetries in their theories, to provide proofs of the probability rule that their observations support!
- Response:
  - Experimenters that get Maverick spin results can still reach the Born rule by considering explanations of natural phenomena e.g. the blue sky.
  - Experimenters that get Maverick spin results and that also live in red-sky worlds might not be able to reach the Born rule at all. But there is no reason to think that worlds with maverick natural processes can even host experimenters.