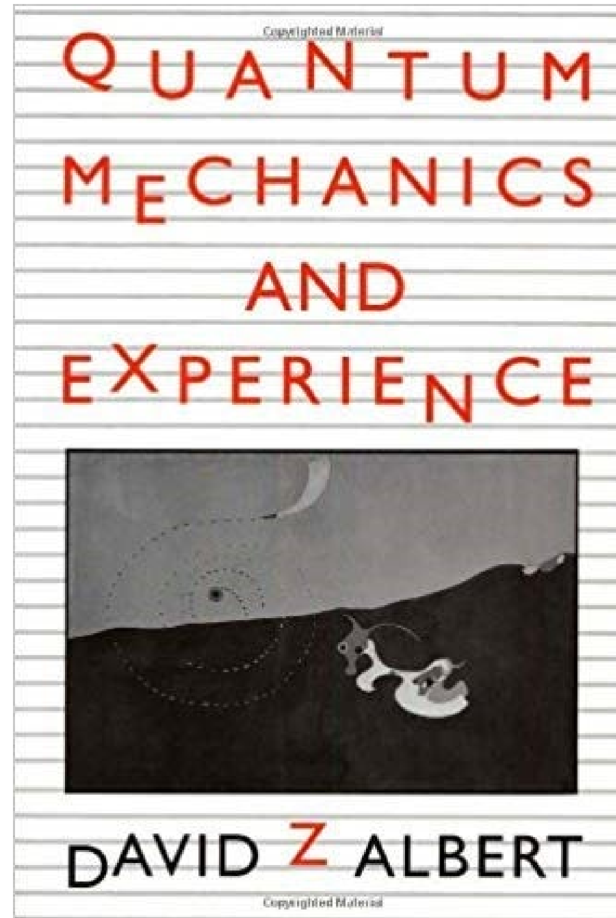
PHYS/PHIL 329 Lecture 15: The collapse of the wave function.

Kelvin McQueen - 3/31/2020

Today's Lecture

- Albert's chapter 5 on the collapse of the wave function.
- What causes collapse?
  - Measurement?
  - Consciousness?
  - Macroscopicness?
- Why is this so hard to experimentally determine?
- The simplest dynamical collapse theory: the GRW interpretation.
- Does GRW guarantee determinate measurement outcomes?



Recap: ψ-epistemic vs. ψ-ontic

- The *ψ-epistemic* view: quantum states (i.e. what state vectors represent) are states of knowledge/belief.

- The *ψ-ontic* view: quantum states are states of reality.
  - Motivations:
    - Interference effects suggest that quantum state components interact.
    - PBR theorem & Hardy's theorem.
    - The miracle argument for scientific realism.
  - *ψ-ontic* views – the big three:
    - *Dynamical collapse theories*.
    - De Broglie-Bohm Theory.
    - Many worlds theories.

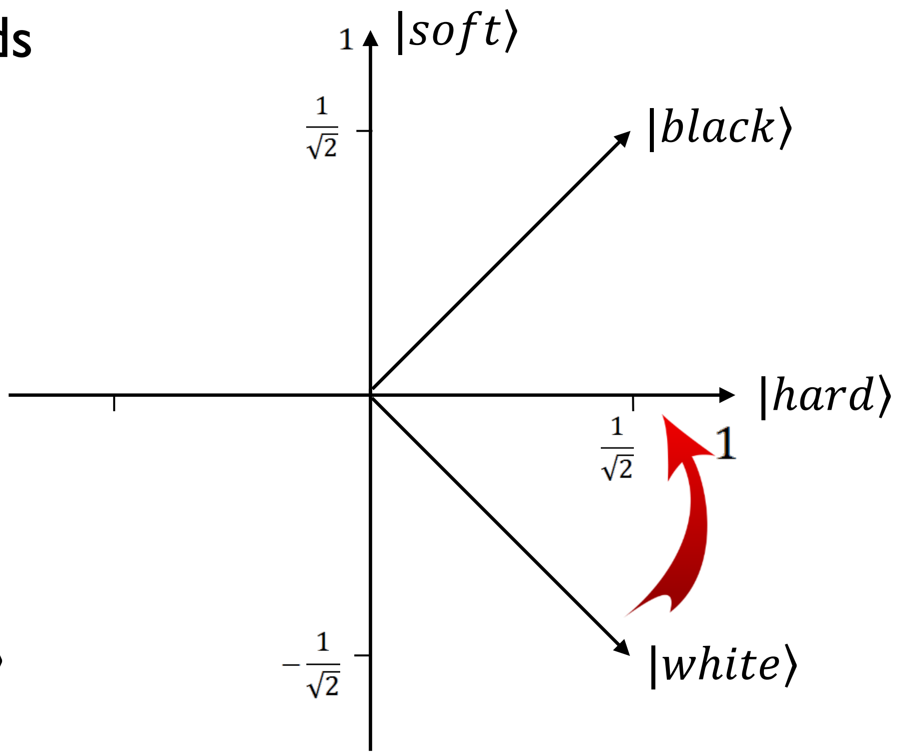
Recap: Collapse of the state vector

- Imagine a *hardness box measurement* finds a *white* electron to be *hard*.
- The measurement *probabilistically collapses* the electron's state vector e.g.
  - Pre-measurement state vector:

$$|white\rangle = \frac{1}{\sqrt{2}}|hard\rangle - \frac{1}{\sqrt{2}}|soft\rangle$$

  - Post-measurement state vector:

$$|hard\rangle = \frac{1}{\sqrt{2}}|black\rangle + \frac{1}{\sqrt{2}}|white\rangle$$



- According to collapse theories:
  - The state vector represents reality (it's a ψ*-ontic* view).
  - So, the collapse of the state vector represents a real physical process.

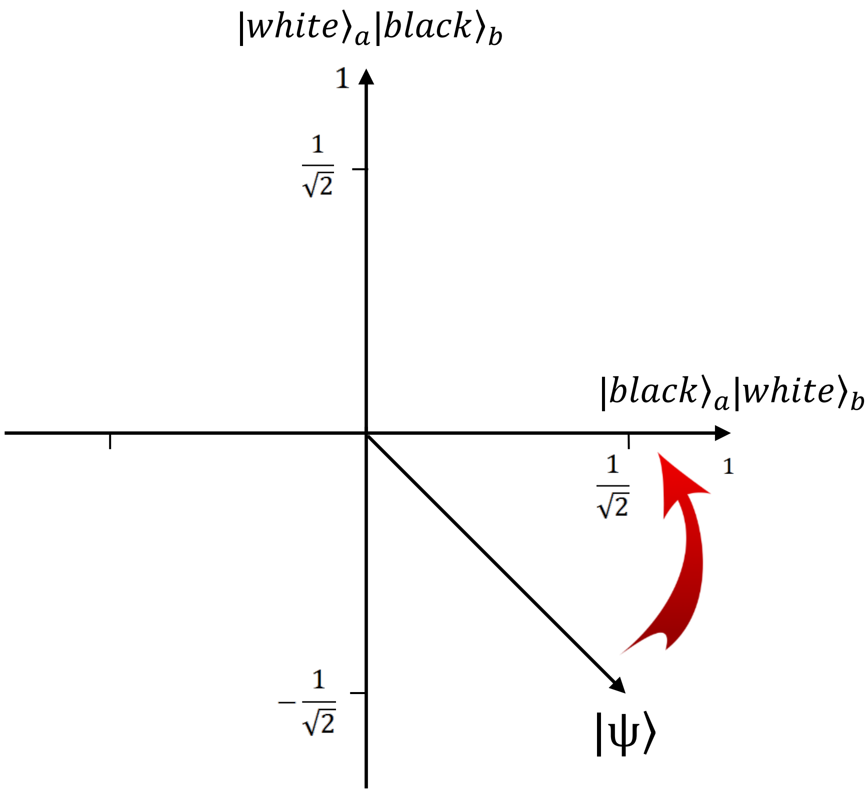
Recap: Remote collapses

- Imagine the initial state of particles a and b is:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|black\rangle_a|white\rangle_b - \frac{1}{\sqrt{2}}|white\rangle_a|black\rangle_b$$

- Imagine Alice performs a *color measurement* on her particle and finds that it is *black*.

- The measurement *probabilistically collapses* the state vector of the two-particle system:
  - Pre-measurement state vector:
    $$\frac{1}{\sqrt{2}}|black\rangle_a|white\rangle_b - \frac{1}{\sqrt{2}}|white\rangle_a|black\rangle_b$$
  - Post-measurement state vector:
    $$|black\rangle_a|white\rangle_b$$



- According to collapse theories:
  - The collapse of the state vector represents a real physical process.
  - So, Alice *non-locally* collapses Bob's particle (Bob's particle can be arbitrarily far away from Alice's).
    - Collapse theorists view Bell's theorem as demonstrating that *reality is nonlocal.*
    - Relativistic *no superluminal signaling* still obeyed.

John von Neumann's (1932) hypothesis

- Measurement collapses the wave function.
- In particular, there are two laws of nature:
  - I. When no measurements are going on, the states of all physical systems invariably evolve in accordance with the linear deterministic dynamics (the Schrödinger equation).
  - II. When there *are* measurements going on, the states of measured system evolve in accordance with the postulate of collapse, *not* in accordance with the linear deterministic dynamics.
- Albert's critique: the measurement problem:
  - What these laws say, depend on the meaning of the word "measurement".
  - But that word has no precise meaning in ordinary language and von Neumann made no attempt to define a precise meaning for it.

Eugene Wigner's (1961) hypothesis

- Consciousness collapses the wave function.

- In particular, there are two laws of nature:
  - I. *Non-conscious systems* invariably evolve in accordance with the linear deterministic dynamics (the Schrödinger equation).
  - II. Conscious systems *also* evolve in accordance with the Schrödinger equation, *unless* the conscious experience of the system is about to superpose, in which case, the system collapses so that its state of consciousness remains in some definite state.
- Albert's critique: the precision problem:
  - What these laws say, depend on the meaning of the word "conscious".
  - But that word has no precise meaning in ordinary language and Wigner made no attempt to define a precise meaning for it.

Another (surprisingly common) hypothesis

- Macroscopicness collapses the wave function.

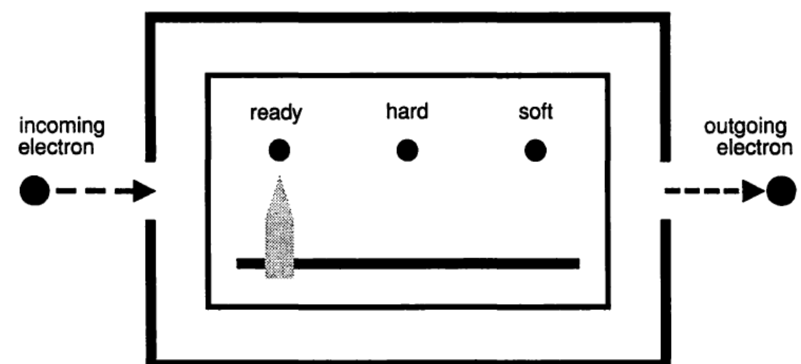
- In particular, there are two laws of nature:
  - I. *Microscopic systems* invariably evolve in accordance with the linear deterministic dynamics (the Schrödinger equation).
  - II. *Macroscopic* systems *also* evolve in accordance with the Schrödinger equation, *unless* a macroscopic property of the system is about to superpose, in which case, the system collapses so that its macroscopic properties remain in definite states.

- Albert's critique (p84):
  - "There is an astonishingly long and bombastical tradition in theoretical physics of formulating these sorts of guesses about precisely when the collapse occurs in language which is so imprecise as to be absolutely useless."
  - "Some of the words that come up in these guesses (besides *measurement* and *consciousness* and *macroscopic*) are *irreversible*, *recording*, *information*, *meaning*, *subject*, *object*, and so on."

Why it's hard to settle the question experimentally

- Why make *guesses* about exactly when and where collapse occurs?
- Why not determine this *by experiment*?
  - The collapse of the state vector corresponds to a real physical event.
  - So, it should have *detectable* consequences.

- How to perform such experiments?
  - Consider two different collapse hypotheses regarding a hardness measurement of a black electron:
    - The *device* hypothesis: the device collapses the state vector.
    - The *brain* hypothesis: the observer's brain collapses the state vector.



Two collapse hypotheses

- *Device hypothesis*: measuring device causes collapse:
  - $|\psi\rangle_{t_1} = |"ready"\rangle_o |"ready"\rangle_m \frac{1}{\sqrt{2}}(|hard\rangle_e + |soft\rangle_e)$
  - $|\psi\rangle_{t_2} = |"ready"\rangle_o |"hard"\rangle_m |hard\rangle_e$ with 0.5 probability
    $OR \quad |"ready"\rangle_o |"soft"\rangle_m |soft\rangle_e$ with 0.5 probability.

- *Brain hypothesis*: observer's brain causes collapse:
  - $|\psi\rangle_{t_1} = |"ready"\rangle_o |"ready"\rangle_m \frac{1}{\sqrt{2}}(|hard\rangle_e + |soft\rangle_e)$
  - $|\psi\rangle_{t_2} = |"ready"\rangle_o \frac{1}{\sqrt{2}}(|"hard"\rangle_m |hard\rangle_e + |"soft"\rangle_m |soft\rangle_e)$
  - $|\psi\rangle_{t_3} = |"ready"\rangle_o |"hard"\rangle_m |hard\rangle_e$ with 0.5 probability
    $OR \quad |"ready"\rangle_o |"soft"\rangle_m |soft\rangle_e$ with 0.5 probability.

Two collapse hypotheses

- Our two hypotheses disagree about the t_2 quantum state:
  - *Device hypothesis*:
    - $|\psi\rangle_{t_2} = |"hard"\rangle_m |hard\rangle_e$ with 0.5 probability
      $OR \quad |"soft"\rangle_m |soft\rangle_e$ with 0.5 probability.
  - *Brain hypothesis*:
    - $|\psi\rangle_{t_2} = \frac{1}{\sqrt{2}}(|"hard"\rangle_m |hard\rangle_e + |"soft"\rangle_m |soft\rangle_e)$

- For example, they disagree about:
  - The position of the pointer.
    - Device hypothesis says it's definite.
    - Brain hypothesis says it isn't.
  - The hardness of the electron.
    - Device hypothesis says it's definite.
    - Brain hypothesis says it isn't.

Two collapse hypotheses

- What are the t_2 predictions for a pointer position measurement ?
  - *Device hypothesis*:
    - The t_2 state is:

$$|\text{"}hard\text{"}\rangle_m |hard\rangle_e \text{ with 0.5 probability}$$
$$OR\ \ |\text{"}soft\text{"}\rangle_m |soft\rangle_e \text{ with 0.5 probability}$$

    - …so the measurement will ***reveal*** that the pointer is either in

$$|\text{"}hard\text{"}\rangle_m \ or\ |\text{"}soft\text{"}\rangle_m.$$

  - *Brain hypothesis*:
    - Since the t_2 state is:

$$\frac{1}{\sqrt{2}}\left(|\text{"}hard\text{"}\rangle_m |hard\rangle_e + |\text{"}soft\text{"}\rangle_m |soft\rangle_e\right)$$

    - …the measurement will ***collapse the pointer's state and then*** reveal it to be:

$$|\text{"}hard\text{"}\rangle_m \ or\ |\text{"}soft\text{"}\rangle_m.$$

- Same results! Pointer position measurements don't help.
  - For the same reason, neither will hardness measurements:
    - *Device hypothesis*: the measurement will *reveal* the electron's hardness.
    - *Brain hypothesis*: the measurement will collapse and then reveal the electron's hardness.

Device hypothesis Vs brain hypothesis

- What's going wrong?
  - According to the brain hypothesis, neither the pointer nor the electron has any definite states, at t_2.
  - So measurements of the pointer or the electron will collapse them into the states that the device hypothesis predicts they are already in.
- This suggests what the *right* experiments are...
  - We need to find a way to measure a state like this:

$$\frac{1}{\sqrt{2}}(|"hard"\rangle_m |hard\rangle_e + |"soft"\rangle_m |soft\rangle_e)$$

  - ...without collapsing it.
- So we need to:
  - Work out what operator this state is an eigenstate of, and what the eigenvalue is.
  - Find a way of reliably measuring that operator, so that we can determine with certainty, whether the system possesses the relevant eigenvalue.
    - If it does, the device hypothesis is falsified.

Why such experiments are difficult

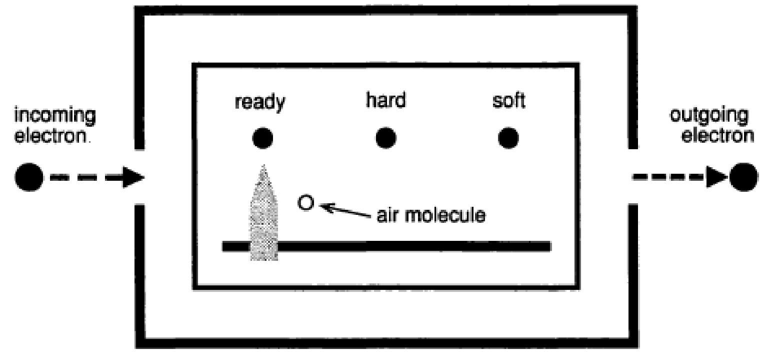
- Imagine we work out that this state:

$$\frac{1}{\sqrt{2}}(|"hard"\rangle_m|hard\rangle_e + |"soft"\rangle_m|soft\rangle_e)$$

- ...is an eigenstate of operator O with eigenvalue e.
  - We now want to measure O at t_2.

- Unfortunately, this is practically impossible.
  - For we must prevent the m+e system from entangling with its environment.
    - If it entangles, then our measurement will collapse the state, but we must avoid collapsing the state.
  - We may never have the technology to actually do this!

An illustration of the difficulty



- Imagine we try to measure O at t_2, but we've let an air molecule into our lab, which sits by the pointer.
- The brain hypothesis then predicts:

$$|\psi\rangle_{t_1} = |by\_"ready"\rangle_{air}|"ready"\rangle_m \frac{1}{\sqrt{2}}(|hard\rangle_e + |soft\rangle_e)$$

$$|\psi\rangle_{t_2} = \frac{1}{\sqrt{2}}(|by\_"hard"\rangle_{air}|"hard"\rangle_m|hard\rangle_e + |by\_"soft"\rangle_{air}|"soft"\rangle_m|soft\rangle_e)$$

- Unfortunately, the t_2 state is no longer an eigenstate of O.
  - So, the O-measurement will collapse the state to one of the possible eigenstates of O with the same probabilities predicted by the device hypothesis.

How to evaluate collapse hypotheses?

- We can ask whether such theories can actually be *precisely formulated* at all.
  - "Device" and "brain" collapse theories are not precisely formulated and offer no collapse *dynamics*.

- Once we've achieved the desired precision we can explore both *experimental* and *theoretical* tools for theory selection.
  - Experimental:
    - We can ask whether the theory
      - makes novel predictions that are feasible to test with foreseeable technology;
      - is bounded in any interesting ways by existing experimental data.
  - Theoretical:
    - We can ask whether the theory
      - is internally coherent;
      - fits with everything else we know about the world;
      - explains what it was supposed to explain;
      - explains what it was supposed to explain better than competing (e.g. no-collapse) theories;

      etc.

Constraints on dynamical collapse

- *Constraint 1*: Guarantee consistency with experiments on isolated particles.
  - Isolated particles don't collapse (often enough to be observed).
- *Constraint 2*: Guarantee that measurements have specific outcomes.
  - No superpositions of measurement devices (or at least, of brains).
- *Constraint 3*: Guarantee that measurements have outcomes *with the right probabilities*.
  - Outcomes must correspond to Born's probability rule.

The Ghirardi-Rimini-Weber (GRW) theory

- The simplest dynamical collapse theory was published in 1985 by three Italian physicists, Giancarlo Ghirardi, Alberto Rimini, and Tullio Weber, collectively known as GRW.
  - Key papers:
    - Ghirardi, G.C., Rimini, A., and Weber, T. (1985). "A Model for a Unified Quantum Description of Macroscopic and Microscopic Systems". *Quantum Probability and Applications*, L. Accardi et al. (eds), Springer, Berlin.
    - Ghirardi, G.C., Rimini, A., and Weber, T. (1986). "Unified dynamics for microscopic and macroscopic systems". *Physical Review* D 34: 470.

- The basic idea is very simple:
  - Every elementary particle possesses a new type of fundamental property:
    - *A probability per unit time for spontaneously collapsing to a definite position.*

E.g. if a particle is in the following state:

$$\frac{1}{2}|X = 5\rangle + \frac{\sqrt{3}}{2}|X = 10\rangle$$

…then there is *an extremely small probability* that the particle will spontaneously collapse to
$X = 5$ with probability 0.25 or
$X = 10$ with probability 0.75.

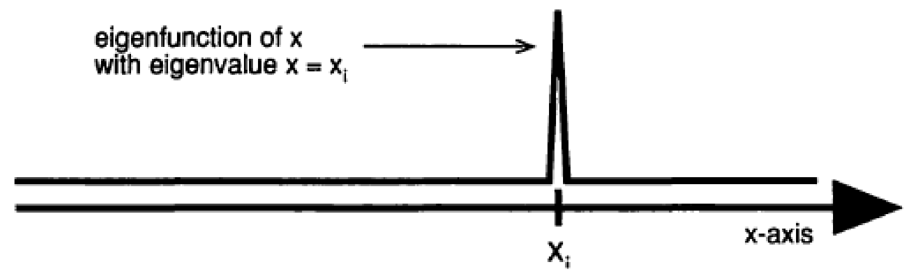
An idealized version of the GRW theory

- Consider a particle confined to the X dimension of space:

$$|A\rangle = a_1|x_1\rangle + a_2|x_2\rangle + a_3|x_3\rangle + \cdots + a_n|x_n\rangle$$

- The particle is stipulated to have *a 10^{-16} probability per second for spontaneously collapsing* to a definite position. So it undergoes "a localization", on average, every hundred million years.

- The collapse has the effect of multiplying all but one of the a_i terms by 0, leaving only one term behind, which gets normalized to one.
  - In other words: the particle's wave function gets multiplied by a position operator eigenfunction.



- The probability that the particle will collapse to $|x_i\rangle$ is given by the familiar (Born) probability rule:

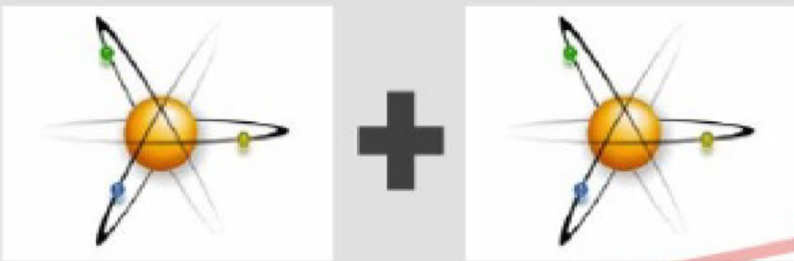
$$(\langle A|x_i\rangle)^2 = (a_i)^2$$

- Notice that *objective probability* now plays *two* roles.

Amplification in idealized GRW

- Now consider what happens to entangled particles. Recall:
  - $|\psi\rangle = \frac{1}{\sqrt{2}}|X_5\rangle_1|X_{55}\rangle_2 + \frac{1}{\sqrt{2}}|X_9\rangle_1|X_{99}\rangle_2$
- When particles are entangled like this, a collapse on either particle equally localizes both.
  - If particle 1 undergoes spontaneous localization, *it's as if its position has been measured*: if particle 1 collapses to position X_9 then particle 2 immediately collapses to position X_{99}.
- *Both* particles 1 and 2 each have a *10^{-16} localization probability per second.*
  - So the probability of *both* particles being localized in a given second has doubled (2x*10^{-16}*).
  - No need to wait 100 million years for collapse, now it's only a 50 million yearlong wait (on average).
- Now consider what happens to ordinary macroscopic objects, e.g. your average cat, which is made of ~10^{27} particles.
  - One of the cat particles will localize in about 10^{-11} seconds.
  - If their positions are all entangled (as with Schrödinger's cat), the whole cat will localize in about 10^{-11} seconds.
  - It is no wonder, then, that when we open the box, we see a definite cat state.

The Schrödinger equation is linear ➔ superposition principle



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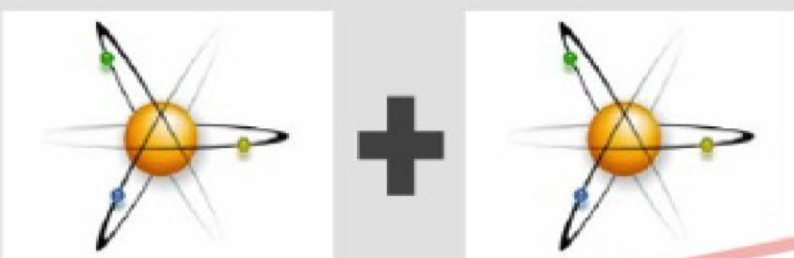
What?

Collapse models ➔ no macroscopic superpositions



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How GRW meet the constraints

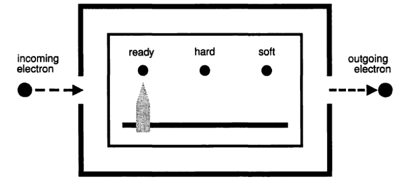
- *Constraint 1*: Guarantee consistency with experiments on isolated particles.
  - GRW: an isolated particle localizes, on average, every hundred million years.

- *Constraint 2*: Guarantee that measurements have specific outcomes.
  - GRW: measurement outcomes are recorded in the positions of macroscopic objects.
  - GRW: via amplification, an average macroscopic object undergoes a localization every 10^{-7} seconds.

- *Constraint 3*: Guarantee that measurements have outcomes *with the right probabilities*.
  - GRW: The probability that a collapse is centred on a given superposition component is given by the familiar (Born) probability rule.

Measurement in the idealized GRW theory

- Returning to this state:



$$|\psi\rangle_{t_1} = |"ready"\rangle_o |"ready"\rangle_m \frac{1}{\sqrt{2}}(|hard\rangle_e + |soft\rangle_e)$$

- ...we can remove o and represent the pointer m in terms of its constituent particles:

$$|\psi\rangle_{t_1} = |X_{r1}\rangle_{m1} |X_{r2}\rangle_{m2} \dots |X_{rn}\rangle_{mn} \frac{1}{\sqrt{2}}(|hard\rangle_e + |soft\rangle_e)$$

- …where $|X_{r2}\rangle_{m2}$ represents the position of particle 2, when the pointer is in the ready position.

- Recall that at t_2, the electron became entangled with the pointer:

$$|\psi\rangle_{t_2} = \frac{1}{\sqrt{2}}(|"hard"\rangle_m |hard\rangle_e + |"soft"\rangle_m |soft\rangle_e)$$

- …which now becomes:

$$|\psi\rangle_{t_2} = \frac{1}{\sqrt{2}}(|X_{h1}\rangle_{m1} |X_{h2}\rangle_{m2} \dots |X_{hn}\rangle_{mn} |hard\rangle_e + |X_{s1}\rangle_{m1} |X_{s2}\rangle_{m2} \dots |X_{sn}\rangle_{mn} |soft\rangle_e)$$

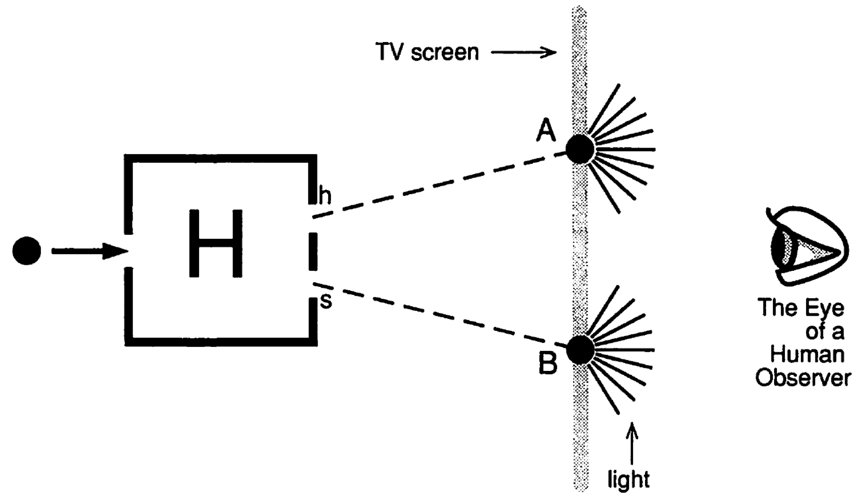
- This state is highly unstable! As soon as one particle collapses, the whole system collapses to:
  - $|\psi\rangle_{t_3} = |"ready"\rangle_o |"hard"\rangle_m |hard\rangle_e$ with 0.5 probability
    OR $|"ready"\rangle_o |"soft"\rangle_m |soft\rangle_e$ with 0.5 probability.

Summary of the basic idea

- *GRW Collapse hypothesis*: elementary particles have a small probability per unit of time for collapsing into a definite position.
- *Amplification*: measuring devices are composed of many entangled particles and so have a high probability per unit time for collapsing into a definite position.
- The probability that a measuring device will collapse into one of its superposition components is given by the Born probability rule.

Experiments with television screens

- Determinate measurement outcomes are usually displayed in the positions of things (e.g. pointers).
- But now consider the following set-up: we send a black electron through a hardness device…
- Hard (soft) electrons affect the *energies* (*not the positions*) of the fluorescent electrons at region A (B).
- Affected electrons then release photons towards a human eye. From A we get an image of Trump, from B, Biden.



- Albert and Vaidman (1988) argue that we only get significant position superpositions in the photons (emission superpositions), but light travels too quickly for GRW collapses to go off.
- So, *where is the GRW collapse supposed to occur?*
- Ghirardi et al (1991) responded:
  - Collapse fails to occur in retinal rods (since less than 105 molecules are affected), but transmission along the nervous signal within the brain, involves the displacement of a number of ions of the order of 10^{12}.