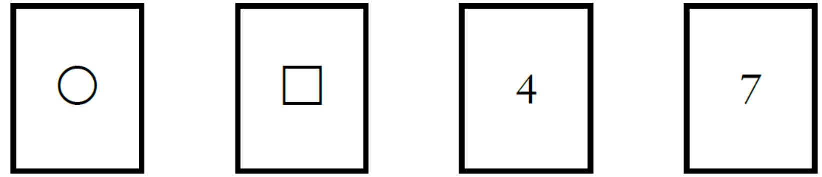

Symbolic logic

Paradox of the Material Conditional

Chapman University. PHIL300. Lectures 25&26. 5/(3&5)/2022.
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Homework 8: Q7

∃x(Fx & Ga), ∀x(Fx → Hx) ⊢ Ga & ∃x(Fx & Hx)

1	(1)	∃x(Fx & Ga)	A
2	(2)	∀x(Fx → Hx)	A
2	(3)	Fa → Ha	2∀E
4	(4)	Fa & Ga	A (for ∃E)
4	(5)	Fa	4&E
2,4	(6)	Ha	3,5→E
2,4	(7)	Fa & Ha	5,6&I
2,4	(8)	∃x(Fx & Hx)	7EI
4	(9)	Ga	4&E
2,4	(10)	Ga & ∃x(Fx & Hx)	8,9&I
1,2	(11)	Ga & ∃x(Fx & Hx)	1,10 ∃E(4)

<u>Incorrect!</u>
Remember, the instantiated name on line (4) for ∃E (on line (11)), cannot be contained on line (10), cannot be contained in lines that (10) depend on [other than the instantial assumption at (4)], and cannot be contained in line (1).

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2	(2)	∀x(Fx → Hx)	A
2	(3)	Fb → Hb	2∀E
4	(4)	Fb & Ga	A (for ∃E)
4	(5)	Fb	4&E
2,4	(6)	Hb	3,5→E
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1,2	(11)	Ga & ∃x(Fx & Hx)	1,10 ∃E(4)

Correct!
By using a name not contained in (1) in the instantial assumption on (4), that is, by using *b* instead of *a*, we ensure that what we derive at (11) does not depend on any special assumptions about b. That's what makes this inference valid.

Homework 8: Q8

~∃x(Fx & Gx) ├ ∀x(~Fx v ~Gx)

1	(1)	~∃x(Fx & Gx)	A
1	(2)	∀x~(Fx & Gx)	1QE
1	(3)	~(Fa & Ga)	2∀E
1	(4)	~Fa v ~Ga	3DM
1	(5)	∀x(~Fx v ~Gx)	4∀I

The paradox of the material conditional

- The *material* (classical logic) conditional
  - False when true/false, true otherwise.

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

- The *indicative* (natural language) conditional
  - Expressed in English by "If.. then…" and variants (see p12).
- The *paradox*:
  - We have strong arguments for *identifying* the indicative conditional with the material conditional (i.e. treating them as *synonymous*).
  - Unfortunately, this identification leads to three *very unintuitive consequences*.

3 unintuitive consequences, these are valid:

- True consequent: $P \vdash Q \rightarrow P$

  (1) Biden won the 2020 election.

  (2) Therefore, if UFOs invade, then Biden won the 2020 election.

- False antecedent: $\sim P \vdash P \rightarrow Q$

  (1) It is false that I am 10 foot tall.

  (2) Therefore, if I am 10 foot tall, then I am invincible.

- Explosion: $P, \sim P \vdash Q$

  (1) Logic is easy.

  (2) Logic is not easy.

  (3) Therefore, the Earth is flat.

Homework 10 (and final exam) questions

- 1. In your own words, explain two reasons for identifying the indicative conditional with the material conditional.
- 2. State the sequents of the three paradoxes, *false antecedent*, *true consequent*, and *explosion*.
- 3. For each of the three sequents, substitute an absurd-sounding English argument, ones not yet discussed.
- 4. Choose one of the three sequents. In your own words, describe the pragmatic explanation of it. Then, say whether you think the problem for that sequent has been solved. *Explain your answer i.e. explain the reasons for why you think this.*

This weeks' lectures has three parts

- Part 1:
  - Why *identify* the indicative and material conditionals in the first place?
- Part 2:
  - How do *classical logicians* make sense of these unintuitive consequences? (Pragmatic explanation)
- Part 3:
  - How do *non-classical logicians* propose to change logic to solve this? (Relevance logic)

Truth-functionality

- A logical connective is *truth-functional* iff the truth-value of a sentence containing the connective *is a function of* the truth values of its component sentences.

- Conjunction is the simplest example:
  - The truth-value of
    - It's raining and it's snowing
  - …is a function of the truth-value of
    - It's raining
  - …and the truth-value of
    - It's snowing.
  - *Truth-tables* capture this functionality:

It's raining	It's snowing	It's raining & it's snowing
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**F**

Truth-functionality and rules of proof

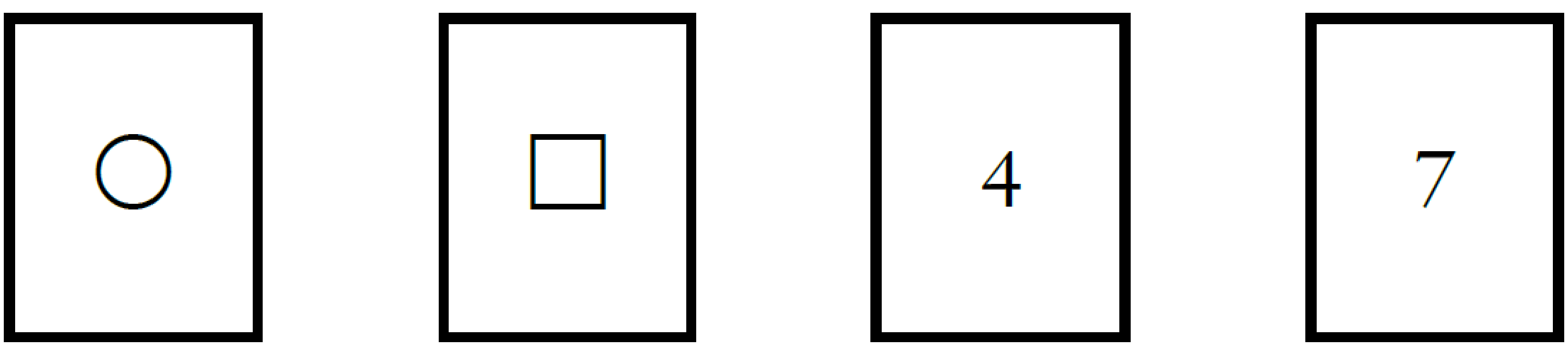
- Truth-tables define introduction and elimination rules.

P	Q	P&Q
T	T	**T**
T	F	**F**
F	T	**F**
F	F	**F**

- Row 1 tells us that if both conjuncts are true, then we can prove the conjunction.
- Rows 2-4 tell us that there is no other situation in which the conjunction can be proved.
  - &-introduction.
- Rows 1-4 tell us that either conjunct can be proved if and only if the conjunction is true.
  - &-elimination.

Is the indicative conditional truth-functional?

- Let's examine this with an exercise.
- Each of the four cards below has a figure on one side and a number on the other side.
- Figures and numbers should appear according to this rule:
  - ***If a card has a circle on one side, then it has an even number on the other.***
- ***Question***. Which of the cards do you need to turn over to find out whether the rule is broken? Choose the card (or those cards) you definitely have to turn over, and only that card (or those cards), in order to determine whether the rule is violated.



Is the indicative conditional truth-functional?

- This is called the *Wason selection test.*
- Studies have found that less than 10% of subjects get the correct solution!

- However:
  - Subjects readily accept the correct (material conditional) solution when it is explained to them.
  - Subjects are more likely to get the correct solution if the example involves a familiar *social* rule.

- The moral of the example:
  - The material conditional "false when true/false" idea seems right.
  - But it need not be obvious at first.

Why the material conditional truth table?

- To try and show that an indicative conditional is false, we try to show that it is possible for the antecedent to be true and the consequent false.
  - This suggests "false when true/false".

P	→	Q
T	**F**	F

- The only possible entries for the remaining rows correspond to the material conditional.

- Suppose we agree on the first two rows. We can then write out all 4 possibilities for the bottom two rows. Clearly, only option (4) is viable, since (1)-(3) already mean something else.

P	Q	(1)	(2)	(3)	(4)
T	T	T	T	T	**T**
T	F	F	F	F	**F**
F	T	T	F	F	**T**
F	F	F	T	F	**T**

Why the material conditional truth table?

- The bottom two rows also explain the *invalidity* of the following fallacious arguments:

(1) P→Q
(2) Q
(3) So, P

(1) F→T
(2) T
(3) So, F

P	→	Q
F	**T**	T

(1) P→Q
(2) ~P
(3) So, Q

(1) F→F
(2) T
(3) So, F

P	→	Q
F	**T**	F

- The full truth table is also consistent with conjunction and disjunction truth tables: P→Q ~(P&~Q) ~P v Q

Why the material conditional truth table?

- Example: *If I am healthy, I will come to class* (H → C).

H	C	H → C
T	T	T
T	F	F
F	T	T
F	F	T

Why the material conditional truth table?

- Example: *If I am healthy, I will come to class* (H → C).

H	C	H → C
T	**T**	**T**
T	F	F
F	T	T
F	F	T

- Row 1: I am healthy and I come to class. I have clearly kept my promise; the conditional is *true*.

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- Example: *If I am healthy, I will come to class* (H → C).

H	C	H → C
T	T	T
T	**F**	**F**
F	T	T
F	F	T

- Row 1: I am healthy and I come to class. I have clearly kept my promise; the conditional is *true*.
- Row 2: I am healthy, but I have decided to stay home and watch television. I have broken my promise; the conditional is *false*.

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- Row 2: I am healthy, but I have decided to stay home and watch television. I have broken my promise; the conditional is *false*.
- Row 3: I am not healthy, but I have come to class anyway. I am coughing and sneezing, and you're not happy about it, but I did not violate my promise; the conditional is *true*.

Why the material conditional truth table?

- Example: *If I am healthy, I will come to class* (H → C).

H	C	H → C
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T	F	F
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- Row 1: I am healthy and I come to class. I have clearly kept my promise; the conditional is *true*.
- Row 2: I am healthy, but I have decided to stay home and watch television. I have broken my promise; the conditional is *false*.
- Row 3: I am not healthy, but I have come to class anyway. I am coughing and sneezing, and you're not happy about it, but I did not violate my promise; the conditional is *true*.
- Row 4: I am not healthy, and I did not come to class. I did not violate my promise; the conditional is *true*.

Summary of arguments for material conditional

- Classical logic's reasons for identifying the indicative conditional with the material conditional:
  - The way we refute indicatives suggests they are truth-functional, since it suggests "false when true/false".
  - This need not be immediately obvious—we can allow that a certain amount of reasoning is needed to see this.
  - If we demand truth-functionality, the material conditional is the only possible truth-table.
  - The material conditional works extraordinarily well for determining the (in)validity of real-life arguments.
  - It is consistent with the rest of classical logic, e.g. we have the equivalences: $P \rightarrow Q$ $\sim(P \& \sim Q)$ $\sim P \vee Q$
  - Many examples of English indicative sentences seem consistent with the truth table.

The paradoxes

P	P→Q	Q
T	**T**	**T**
T	F	F
F	**T**	**T**
F	**T**	F

- Looking at the truth table, we see that
  - If the consequent is true, the conditional is guaranteed to be true.
  - If the antecedent is false, the conditional is guaranteed to be true.
- This is reflected in two of our derived rules (p29):

S17*	$P \vdash Q \rightarrow P$	True Consequent
S18*	$\sim P \vdash P \rightarrow Q$	False Antecedent

- These sequents are easily proved using arrow introduction.
- But when we substitute English sentences in for P and Q, we get very strange results.

The paradoxes illustrated

- True consequent:
  - (1) Life exists on Earth.
  - (2) Therefore, if life exists on other planets, then life exists on Earth.
- False antecedent:
  - (1) It is false that the moon is made of cheese.
  - (2) Therefore, if the moon is made of cheese, then life exists on other planets.

- Intuitively, are these valid or invalid?
- Try to think of your own illustrations. Try to make them sound as absurd as possible.

The explosion paradox

- The following is valid in classical logic:

$P, \sim P \vdash Q$

1	(1)	P	A
2	(2)	~P	A
3	(3)	~Q	A (for RAA)
1,2	(4)	Q	1,2 RAA(3)

- An English substitution:
  - Logic is easy, logic is not easy, therefore the Earth is flat.

Solutions

- Solutions can be distinguished into two strategies:

(1) Hold on to classical logic (especially the material conditional) and try to "explain away" the paradoxes.

- The pragmatic account.

(2) Abandon classical logic, and try to construct a new logic for conditionals, and related parts of logic.

- Relevance logic

    - Exam/homework will concern (1).

Homework 10 (and final exam) questions

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The pragmatic account

- Everyday conversation is governed by *conventions* or *rules*. Here are three conversational conventions:
  - *Quality*: speak truthfully, reason validly.
  - *Relevance*: be relevant.
  - *Quantity*: give just the right amount of information.
- One can speak truly (satisfying *quality*) yet still *mislead* their audience by violating *relevance* and/or *quantity*.
  - Example 1: Someone asks where John is. Knowing he is at the pub, I say: "*John is at the pub* ***or*** *he is at the library*".
  - Example 2: "*You won't eat those* ***and*** *live*" I say of some (edible) mushrooms, knowing that you will now leave them alone.
- The pragmatic account:
  - The paradoxes are certainly odd, but not because they violate *quality*. They are odd because they violate *quantity and relevance*.

The pragmatic explanation of *true consequent*

- The *true consequent* paradox (P ⊢ Q→P):
  - (1) Biden won the 2020 election.
  - (2) Therefore, if UFOs invade, then Biden won the 2020 election.

- Pragmatic explanation:
  - In conversation, if Q→P is asserted *only* on the basis of P being true, then the assertion violates *quantity*.
  - A listener who wrongly assumes *quantity* is being obeyed, will be *misled* into thinking there is some *relevant connection* between Q and P, creating a violation of *relevance*.

- Examples:
  - After some research, I conclude that Biden won the 2020 election. I then assert, "*If UFOs invade, then Biden won the 2020 election*".
  - The museum guard says, "*If you give me five dollars, then I will let you into the museum*", when, in fact, he will admit you in any case.

The pragmatic explanation of *false antecedent*

- The false antecedent paradox ($\sim P \vdash P \rightarrow Q$):
  - (1) It is false that I am 10 foot tall.
  - (2) Therefore, if I am 10 foot tall, then I am invincible.

- Pragmatic explanation:
  - In conversation, if $P \rightarrow Q$ is asserted *only* on the basis of P being false, then the assertion violates *quantity*. And the addition of Q violates *relevance*.
  - In real-life conversation *we actually use this*, to imply that we think P is false.

- Examples:
  - "If Congress passes serious campaign finance reform, then I'm a monkey's uncle!"
  - "I've a right to think," said Alice sharply, for she was beginning to feel a little worried. "Just about as much right," said the Duchess, "as pigs have to fly ..." — *Alice in Wonderland (1865)*.

The pragmatic explanation of *explosion*

- The explosion paradox (P, ~P ⊢ Q):
  - Logic is easy, logic is not easy, therefore the Earth is flat.
- Logic *teaches us* that in a world where contradictions are true, anything goes!
- Moreover, asserting a contradiction seems to violate all three rules, *quantity*, *relevance*, and even *quality*.
- An alternative proof:

1	(1)	P	A
2	(2)	~P	A
1	(3)	P v Q	1vI
1,2	(4)	Q	2,3vE

Homework 10 (and final exam) questions

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Relevance logic

- Introduces new notation for combining assumption numbers in the assumption columns of proofs:
  - comma (,) versus semi-colon (;).
  - "1, 2" is the result of merely *collecting up* assumptions 1 and 2 and *pooling* them together,
  - "1;2" is the result of *applying* 1 to 2 so that they *interact* to produce conclusions.

- &-intro uses comma:

1	(1) P	A
2	(2) Q	A
1,2	(3) P&Q	1,2&I

- arrow-elim uses semi-colon:

1	(1) P→Q	A
2	(2) P	A
1;2	(3) Q	1,2→E

Relevance logic

- Now compare the following two uses of arrow-intro:

- P→Q, Q→R ⊢ P→R

1	(1) P→Q	A
2	(2) Q→R	A
3	(3) P	A(→I)
1;3	(4) Q	1,3→E
1;2;3	(5) R	2,4→E
1;2	(6) P→R	5→I(3)

- Q ⊢ P→Q

1	(1) Q	A
2	(2) P	A(→I)
1	(3) P→Q	1→I(2)

Solution: new condition on →I

In the assumption set of the consequent (red), the antecedent (blue) must appear and must be semi-colon related to another line number.

Q ⊢ P→Q *is therefore invalid.*

Relevance logic

- It should be noted that nobody has been able to construct a *complete* relevance logic, which all relevance logicians agree upon.

- The main problem is that to render explosion invalid:

1	(1)	P	A
2	(2)	~P	A
1	(3)	P v Q	1vI
1,2	(4)	Q	2,3vE [extra condition not satisfied]

- …one thereby renders disjunctive syllogism invalid:

1	(1)	P v Q	A
2	(2)	~P	A
1,2	(4)	Q	1,2vE [extra condition not satisfied]

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