Symbolic logic

The Sorites Paradox

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Homework 8 (Q1-Q6)

Q1: Translate Argument 1:

- All acts that maximize good consequences are ethical.
- Some actions that punish the innocent maximize good consequences.
- Therefore, Some actions that punish the innocent are ethical.

Q2: Translate Argument 2:

- Actions that punish the innocent are not ethical.
- Some actions that punish the innocent maximize good consequences.
- Therefore, Some actions that maximize good consequences are unethical.

Q3: Prove Argument 1.

Q4: Prove Argument 2.

Q5: Prove: $\forall x{\sim}Fx \vdash {\sim}\exists xFx$

Q6: Prove: $\forall x(Fx \rightarrow \exists yLxy), \exists x(Fx \ \& \ Gx) \vdash \exists x\exists y(Gx \ \& \ Lxy)$

Homework 8: Q7

∃x(Fx & Ga), ∀x(Fx → Hx) ├ Ga & ∃x(Fx & Hx)

1	(1)	∃x(Fx & Ga)	A
2	(2)	∀x(Fx → Hx)	A
2	(3)	Fa → Ha	2∀E
4	(4)	Fa & Ga	A (for ∃E)
4	(5)	Fa	4&E
2,4	(6)	Ha	3,5→E
2,4	(7)	Fa & Ha	5,6&I
2,4	(8)	∃x(Fx & Hx)	7EI
4	(9)	Ga	4&E
2,4	(10)	Ga & ∃x(Fx & Hx)	8,9&I
1,2	(11)	Ga & ∃x(Fx & Hx)	1,10 ∃E(4)

<u>Incorrect!</u>
Remember, the instantiated name on line (4) for ∃E (on line (11)), cannot be contained on line (10), cannot be contained in lines that (10) depend on [other than the instantial assumption at (4)], and cannot be contained in line (1).

Homework 8: Q7

∃x(Fx & Ga), ∀x(Fx → Hx) ⊢ Ga & ∃x(Fx & Hx)

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2	(2)	∀x(Fx → Hx)	A
2	(3)	Fa → Ha	2∀E
4	(4)	Fa & Ga	A (for ∃E)
4	(5)	Fa	4&E
2,4	(6)	Ha	3,5→E
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Homework 8: Q7

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1	(1)	∃x(Fx & Ga)	A
2	(2)	∀x(Fx → Hx)	A
2	(3)	Fb → Hb	2∀E
4	(4)	Fb & Ga	A (for ∃E)
4	(5)	Fb	4&E
2,4	(6)	Hb	3,5→E
2,4	(7)	Fb & Hb	5,6&I
2,4	(8)	∃x(Fx & Hx)	7EI
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2,4	(10)	Ga & ∃x(Fx & Hx)	8,9&I
1,2	(11)	Ga & ∃x(Fx & Hx)	1,10 ∃E(4)

<u>Correct</u>!
By using a name not contained in (1) in the instantial assumption on (4), that is, by using <u>*b*</u> instead of <u>*a*</u>, we ensure that what we derive at (11) does not depend on any special assumptions about b. That's what makes this inference valid.

Homework 8: Q8

~∃x(Fx & Gx) ├ ∀x(~Fx v ~Gx)

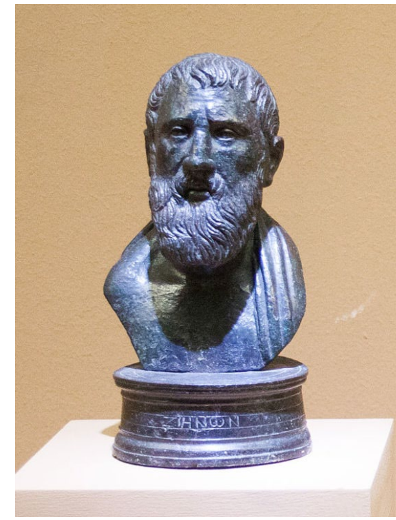
1	(1)	~∃x(Fx & Gx)	A
1	(2)	∀x~(Fx & Gx)	1QE
1	(3)	~(Fa & Ga)	2∀E
1	(4)	~Fa v ~Ga	3DM
1	(5)	∀x(~Fx v ~Gx)	4∀I

Homework (and final exam) questions

- 1. Formulate a Sorites argument with a vague predicate, one not yet discussed in class.
- 2. Explain what is paradoxical about the Sorites argument you have formulated.
- 3. Try to give an example of a non-vague predicate, one that has not yet been discussed in class. Explain why you think it is non-vague.
- 4. Try to come to a conclusion about what is the best solution to the Sorites paradox (you may even try to add your own solution here). In a brief paragraph, explain why you think it is the best solution (for example, explain why you think its problems are less severe than the problems facing the other solutions).

The sorites paradox: origin

- The *sorites paradox* was first formulated (as far as we know) by Eubulides of Miletus in the 4th century BC.



- Despite being ancient and simple to state, there is *still* no agreement on how to solve it, and some think it forces radical revisions to symbolic logic.

- *Informal statement of the paradox*: a single grain of sand is not a heap. Nor is the addition of a single grain of sand enough to transform a non-heap into a heap. And yet we know that at some point we will have a heap.

The sorites paradox: formal statement

- The paradox can be stated using repeated applications of arrow elimination (→E) a.k.a. modus ponens:

  - 1 grain of sand does not make a heap.
  - If 1 grain of sand does not make a heap, then 2 grains don't.
  - If 2 grains of sand does not make a heap, then 3 grains don't.
  - …
  - If 999,999 grains of sand don't make a heap, then 1 million grains don't.
  - Therefore, 1 million grains of sand don't make a heap.

Other illustrations

- The Sorites paradox has *nothing to do with heaps of sand*. Just about any property can illustrate it:

  - Someone who is 7 feet in height is tall.
  - If someone who is 7 feet in height is tall, then someone 6'11.9 in height is tall.
  - If someone who is 6'11.9 in height is tall, then someone 6'11.8 in height is tall.
  - ….
  - Therefore, someone who is 3 foot in height is tall.

Other illustrations

- The Sorites paradox has nothing to do with heaps or heights. Just about any property can illustrate it:
  - A man with 1 hair on his head is bald.
  - If a man with 1 hair on his head is bald, a man with 2 hairs on his head is bald.
  - If a man with 2 hairs on his head is bald, a man with 3 hairs on his head is bald.
  - ….
  - Therefore, a man with 100,000 hairs on his head is bald.

What's common to all illustrations?

- 'Heap', 'tall', 'bald', are all *vague* predicates.
- Vague predicates have *definite cases of application* and *definite cases of non-application*.
  - Someone who is 7 foot tall is *definitely* tall.
  - 1 grain of sand is *definitely not* a heap of sand.
  - Colin Kaepernick is *definitely not* bald.

- Vague predicates also admit *borderline cases* i.e. cases where there appears to be *no fact of the matter* as to whether the predicate applies, sometimes called "grey areas".
  - A man who is 5.11 is *borderline tall.*
  - Three pinches of sand is a *borderline heap*.
  - George Costanza is *borderline bald*.

- Question: are there any non-vague predicates in English?

Finding a solution

- The paradox relies on only three assumptions.
  - There is the *initial premise*:
    - E.g. a man with 1 hair on his head is (definitely) bald.
  - Then there is the "*sorites premise*":
    - E.g. for any number n, if someone with n hairs on his head is bald, then someone with n +1 hairs on their head is bald.
  - Then there is the assumption that *the argument is valid*—which is supposedly confirmed by symbolic logic:
    - $P \rightarrow Q, P \vdash Q$ (→E)

Finding a solution

- There is no agreed-upon solution to the sorites paradox. Proposed solutions fall into three categories:
  - Solution 1: Deny the initial premise.
    - Nihilism
  - Solution 2: Deny the Sorites premise.
    - 2.1: the epistemic view
    - 2.2: truth-value gaps
  - Solution 3: Deny the validity of the argument.
    - Continuum-valued logic

Solution 1: deny the initial premise

- The idea is that there is something *defective* about vague predicates.
  - They don't really apply to the world.
  - Only *absolutely precise* language applies to the world
  - (Symbolic logic deals with precise language just fine.)
  - Will appeal to philosophers who think there exists nothing but atoms in the void! (*Nihilism*)

- Problems:
  - Very radical consequences: there are no bald men; there are no tall people, there are no heaps of sand, etc.
  - Vague language is ubiquitous and isn't going away. <u>We reason with it all the time</u>, so logic had better deal with it somehow.

Solution 2: deny the Sorites premise

- Solution 2.1: the epistemic view
  - The idea is that there are no borderline cases.
    - There *is* a sharp-cut off point for heaps, tallness, and baldness.
    - We just *don't know* where they are!
  - Problems:
    - Entails that a statement such as the following must be *false*:

      If a man with 1002 hairs on his head is bald, then a man with 1003 hairs on his head is bald.
    - Men with 1003 hairs on their head must take care! (which seems absurd).
    - How could we not know facts like this?
    - Presumably, predicates like 'bald' get their meanings from how we use them. Hard to see how our usage could create unknowable sharp cut-offs.

Solution 2: deny the Sorites premise

- **Solution 2.2: truth-value gaps**
  - The idea is that (classical) symbolic logic is wrong to think there are only two truth values (true and false). There is also "undefined".

    "Colin is bald" – false.

    "Shaq is bald" – true.

    "George is bald" – undefined.

    - Consequently, not all of the Sorites premises will be true, and so the Sorites argument is unsound.

  - Problems:
    - Imagine a borderline case of a rainy day, and someone says, "Either it is raining or it is not raining", and "If it is raining, then it is raining"—neither will be true.
    - "If George is bald, then with one less hair he would still be bald"—If George is borderline, how do we evaluate this sentence?
    - The problem of *higher-order vagueness*: where to draw the line between undefined and defined?

Solution 3: deny the validity of the argument

- The idea is that *(classical) symbolic logic is wrong* to think there are only two truth values (true and false). Rather, truth *comes in degrees*.
  - For example, we might have the following evaluations:
    - "A 7' tall man is tall for an adult person"—true to degree 0.95.
    - "A 6'11 tall man is tall for an adult person"—true to degree 0.94.
  - Now consider the Sorites premise:
    - If a 7' tall man is tall for an adult person, then a 6'11 tall man is tall for an adult person.
  - The conditional moves from a "more true" statement to a "less true" statement. So the conditional itself cannot be 100% true!
  - This is the basis of *continuum-valued logic.*

Solution 3: deny the validity of the argument

- How does this help? Continuum-valued logic introduces many new rules. For example, the *degree of truth* of the following sentences might be defined as:
  - P&Q = min(P, Q).
  - PvQ = max(P, Q).
  - ~P = 1 minus P.
  - P→Q = 1 minus (P minus Q).
  - Hence, the conditional:

    If a 7' man is tall for an adult person, then a 6'11 man is tall for an adult person.
  - …has *degree of falsity* given by 0.95 - 0.94 = 0.01.
- Finally, the idea is that we can introduce another rule, stating that when conditionals are stacked up together (as in the Sorites), *degree of falsity* gets *added* each time.
- Then the Sorites argument moves from *very true* to *very false*—a new form of invalidity.

Solution 3: deny the validity of the argument

- Here's an example. We start with the following assignments:
  - A is bald: 0.9
  - B is bald: 0.7
  - C is bald: 0.5
  - D is bald: 0.3
- We then construct the Sorites:

A is bald.	[0.9]
If A is bald then B is bald.	[0.8]
If B is bald then C is bald.	[0.8]
If C is bald then D is bald.	[0.8]
So, D is bald.	[0.9-0.2-0.2-0.2=0.3]

- We end up with an argument that has true premises (greater than 0.5) but a false conclusion (less than 0.5).
- The Sorites argument is invalid!

Solution 3: deny the validity of the argument

- Problem: no one has yet developed a fully satisfactory general continuum-valued logic.
  - Let the following sentence have 0.5 degree of truth:
    - It is raining.
  - Its negation presumably then also has 0.5 degree of truth:
    - It is not raining.
  - What, then, is the degree of truth of these statements:
    - It is raining and it is raining.
    - It is raining and it is not raining.
  - Seems like they have the same truth value (true to degree 0.5).
  - But one is a contradiction!

Further reading on Sorites paradox

- Lectures upon which these slides were based:
  - https://www3.nd.edu › ~jspeaks › courses › _HANDOUTS › sorites
- Stanford Encyclopedia of Philosophy:
  - https://plato.stanford.edu › entries › sorites-paradox

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