

# Philosophy of quantum mechanics

VU University Amsterdam: W\_MASP\_TF013 Lecture 2: 5/2/2015

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# Online QM videos

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- ▶ The double-slit experiment:
  - ▶ <https://www.youtube.com/watch?v=DfPeprQ7oGc>
- ▶ Debate between David Albert (collapse), Sean Carroll (many-worlds), Sheldon Goldstein (additional variables), and Ruediger Shack (anti-realist).
  - ▶ <http://www.worldsciencefestival.com/2014/06/measure-measure-can-reconcile-waves-particles-quantum-mechanics/>
- ▶ Debate between Sean Carroll (many-worlds) and David Albert (critic).
  - ▶ <http://bloggingheads.tv/videos/1728>
- ▶ David Chalmers on the hard problem of consciousness.
  - ▶ [http://www.ted.com/talks/david\\_chalmers\\_how\\_do\\_you\\_explain\\_consciousness?language=en](http://www.ted.com/talks/david_chalmers_how_do_you_explain_consciousness?language=en)

# Today's Lecture

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- ▶ Recap from previous lecture.
- ▶ Discussion of the first half of Albert's (1992) chapter two: "The Mathematical Formalism".
- ▶ The mind-body problem.

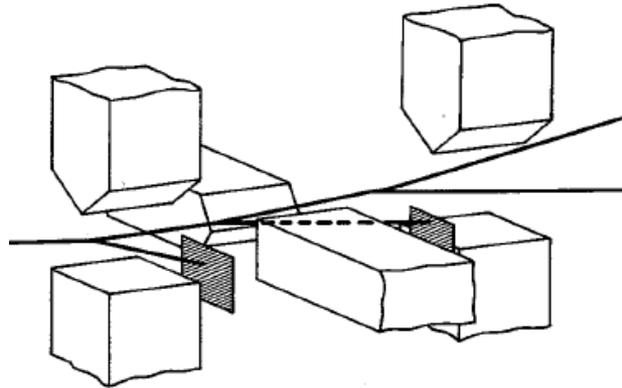


Recap from previous lecture

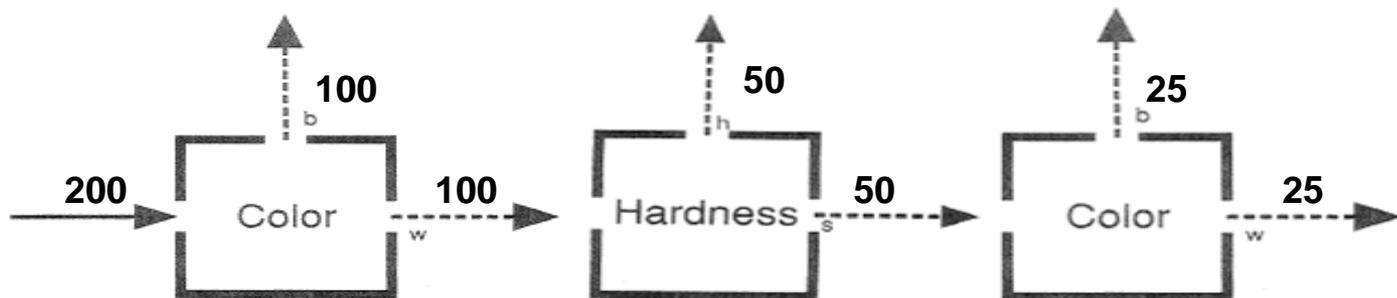


# Idealised quantum mechanics

- ▶ Albert describes idealised versions of real experiments, enabling us to focus on the philosophically interesting aspects.



- ▶ For example, the above version of the Stern-Gerlach experiment becomes the 3-box experiment...



# The need for new (non-classical) concepts

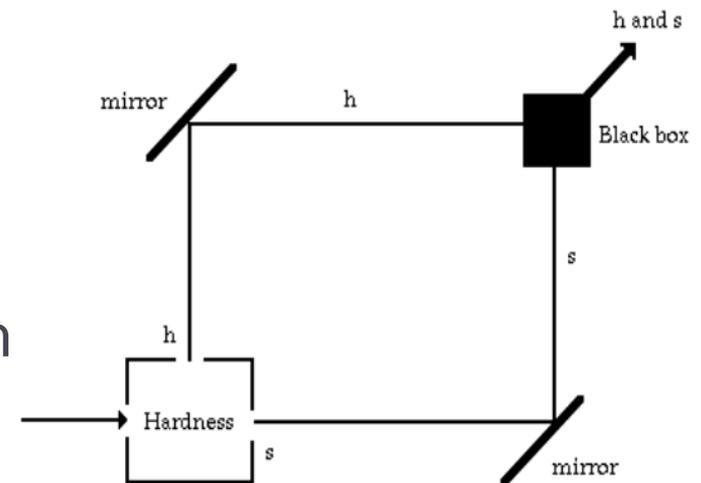
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## ▶ The 3-box experiment.

- ▶ After finding a collection of electrons to be white (1<sup>st</sup> box) and hard (2<sup>nd</sup> box), we expect the third (colour) box to confirm that all electrons are white.
  - ▶ Instead we get 50/50 results so hardness measurements “randomise” colour (and vice versa).

## ▶ The 2-path experiments 1 & 2.

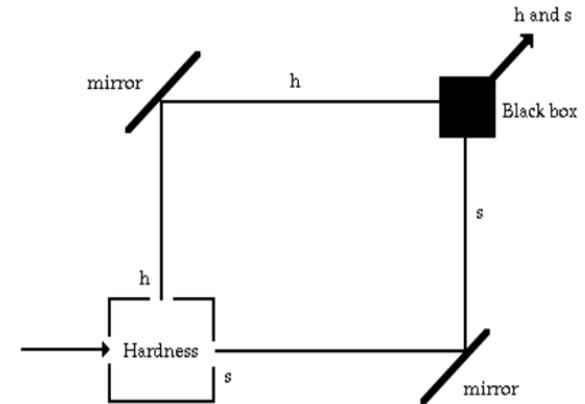
- ▶ This apparent randomisation was confirmed by (1) sending through white electrons and measuring hardness and (2) sending through hard electrons and measuring colour.



# The need for new (non-classical) concepts

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- ▶ The 2-path experiments 3 & 4
  - ▶ But “randomisation” alone could not explain the results of (3) sending through white electrons and measuring colour. Why do we get all white?
  - ▶ And (4) *just* adding a wall to the s-path *does* yield randomisation. This is baffling!



- ▶ Whatever was happening, we realised we couldn't adequately explain these results in terms of any logical possibility given in classical terms.
- ▶ So we introduced “superposition” as a term for the non-classical state of electrons in such experiments.
- ▶ Now we need a theory of superposition states.

# Constraints on formulating the theory

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- ▶ Ideally, we need a mathematical description that:
  - ▶ *Predicts* the results of all of these experiments in terms of superpositions (and perhaps other quantum properties).
  - ▶ *Explains* why these results occur in terms of superpositions.
  - ▶ *Describes* the real nature of superposed entities so that we can understand quantum reality.
- ▶ A further constraint is that the theory must:
  - ▶ Formalise randomisation and the apparent incompatibility of properties like colour and hardness.
    - ▶ Recall: a definite colour entails superposed hardness and vice versa.

# Constraints on formulating the theory

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- ▶ We will see that quantum mechanics goes a long way regarding the *predictive* and *explanatory* constraints.
- ▶ You can make up your own mind about the *descriptive* constraint.
- ▶ By the end of today's lecture you should grasp the formalisation of superposition and incompatibility.
- ▶ We will apply the formalism to the experiments next week.

# Background to “The Mathematical Formalism”

Albert (1992: ch 2)

# Basic structure of chapter 2

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- ▶ Albert builds up to “the five principles of quantum mechanics”.
  - ▶ He provides the basic mathematical background required to understand these principles.
    - ▶ Vectors
    - ▶ Operators
    - ▶ Eigenvectors
- ▶ After discussing how the five principles can be applied Albert adds some further technical details.
  - ▶ Complex vector spaces, Hermitian operators, composite systems etc.
- ▶ Albert then applies the framework to the two-path experiments.

# The five principles

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- ▶ **Physical states**
  - ▶ Represented by certain kinds of vectors.
- ▶ **Measureable properties**
  - ▶ Represented by certain kinds of operators.
- ▶ **Dynamics**
  - ▶ Schrödinger equation changes vector direction over time, deterministically and linearly.
- ▶ **The connection with experiment**
  - ▶ The eigenstate-eigenvalue link: a state possesses the value  $v$  of a property represented by operator  $O$  if and only if that state is an eigenstate of  $O$  with eigenvalue  $v$ .
- ▶ **Collapse**
  - ▶ Measurement of the property represented by operator  $O$  when the measured system is not in an eigenstate of  $O$  will collapse the system into such an eigenstate, with a certain probability.



# Discussion of The Mathematical Formalism



# The plan

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- ▶ Similar to Albert I will split this discussion up as follows:
  - ▶ Vectors
  - ▶ Operators
  - ▶ Eigenvectors
  - ▶ The five principles
- ▶ I will then discuss how to apply the principles, how they relate to the measurement problem, and the Copenhagen interpretation underlying the standard way of thinking about the principles.

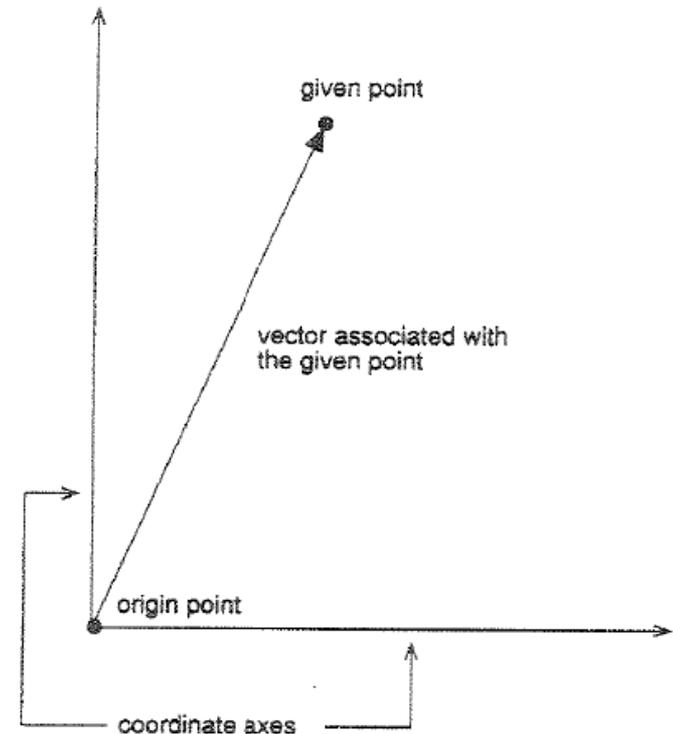


# Vectors

# Vectors and coordinate systems

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- ▶ A vector is a mathematical object with direction and length (or magnitude).
- ▶ To describe a vector's direction and length we need coordinate systems (or *bases*).
- ▶ Coordinate systems are constructed by:
  - ▶ (i) Finding a point (in the space under consideration) and stipulating it to be the origin point &
  - ▶ (ii) defining coordinate axes that stem from the origin point. The number of axes is equivalent to the dimensionality of the space.



# Dimensionality of a vector space

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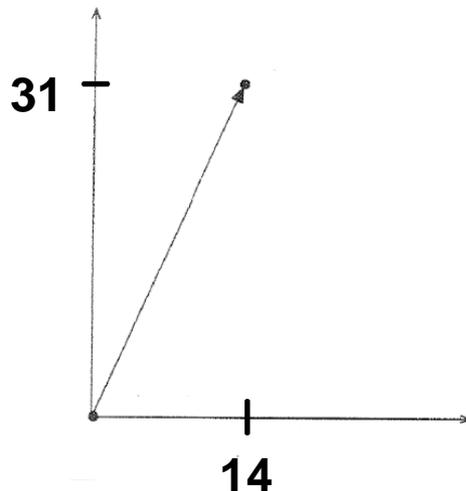
- ▶ Dimensionality of a vector space equals the number of mutually orthogonal (perpendicular) directions in which vectors in that space can point.
- ▶ The previous example is in two-dimensional space. This space is called  $\mathbb{R}^2$  because its points are specified by two real numbers.
- ▶ We appear to live in  $\mathbb{R}^3$
- ▶ We can't depict but can mathematically handle  $\mathbb{R}^{23}$
- ▶ Quantum mechanics often uses complex spaces such as  $\mathbb{C}^2$  whose points are specified by 2 complex numbers (though we can ignore these for our purposes).

# Geometrical/arithmetical representations

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- ▶ We've so far given a geometric representation of vectors.
- ▶ We can also give an arithmetical representation.
  - ▶ Impose a coordinate system on the space and refer to each vector by the coordinates of its tip, as a column.

**Geometrical**



**Arithmetical**

$$\begin{bmatrix} 14 \\ 31 \end{bmatrix}$$

# Notation for vectors

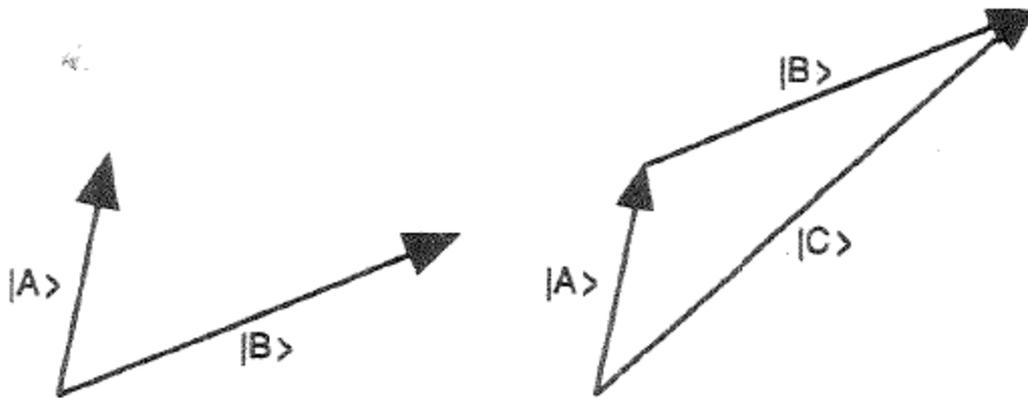
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- ▶ Following Albert (and most of the QM lit.) if we are referring to vector  $A$  then we write:  $|A\rangle$
- ▶ The “ $|\rangle$ ” signifies a vector.
  - ▶ “Dirac” or “Bra-ket” notation.
  - ▶ Named after physicist Paul Dirac.

# Vector addition (geometrical)

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- ▶ To add vectors  $|A\rangle$  and  $|B\rangle$  in geometric representation we:
  - ▶ Move  $|B\rangle$  so that its tail coincides with the tip of  $|A\rangle$  without altering length or direction; and
  - ▶ Find the vector whose tail coincides with the tail of  $|A\rangle$  and whose tip coincides with the tip of  $|B\rangle$  (given its new position).



# Vector addition (arithmetical)

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- ▶ Adding vectors  $|A\rangle$  and  $|B\rangle$  in arithmetical representation works as follows:

- ▶ If  $|A\rangle = \begin{bmatrix} x \\ y \end{bmatrix}$  and  $|B\rangle = \begin{bmatrix} x' \\ y' \end{bmatrix}$

- ▶ Then  $|A\rangle + |B\rangle = \begin{bmatrix} x + x' \\ y + y' \end{bmatrix}$

# Importance of vector addition

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- ▶ Why represent physical states with vectors?
  - ▶ We want to represent incompatibility relations.
    - ▶ E.g. definite hardness entails colour superposition.
- ▶ A given vector is always equivalent to a sum of two other vectors.
  - ▶ This feature can be used to represent property incompatibility.

# Vector (scalar) multiplication

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- ▶ Vectors can be multiplied by numbers.
- ▶ For example 3 times  $|A\rangle$ , or  $3|A\rangle = |A\rangle + |A\rangle + |A\rangle$ .
- ▶ Geometrical representation:
  - ▶  $3|A\rangle$  yields the vector that is three times the length of  $|A\rangle$  that points in the same direction.
- ▶ Arithmetical representation:
  - ▶  $3|A\rangle = \begin{bmatrix} 3x \\ 3y \end{bmatrix}$  (assuming  $|A\rangle$  is in a 2D space).

# Vector (vector) multiplication

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- ▶  $|A\rangle$  times  $|B\rangle$  will be written  $\langle A|B\rangle$ .
- ▶ (Vector multiplication is also known as the *inner product* or *dot product* of two vectors.)
- ▶ In arithmetical representation:
  - ▶ If  $|A\rangle = \begin{bmatrix} x \\ y \end{bmatrix}$  and  $|B\rangle = \begin{bmatrix} x' \\ y' \end{bmatrix}$
  - ▶ then  $\langle A|B\rangle = xx' + yy'$
  - ▶ That is, find the product of the x-coordinates, find the product of the y-coordinates, then find the sum of these two values.

# The length of a vector

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- ▶ Useful for determining *vector length*.
  - ▶ The length (or norm) of  $|A\rangle$  is written  $|A|$ .
- ▶ The formula is:

$$|A| = \sqrt{\langle A|A \rangle}$$

- ▶ Consider the first vector we considered (slide 17)...
- ▶ Its length is:

$$\begin{aligned} |A| &= \sqrt{14 \times 14 + 31 \times 31} \\ &= \sqrt{196 + 961} \\ &= \sqrt{1157} \\ &= 34.01 \end{aligned}$$

# Vector (vector) multiplication

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- ▶ In some cases  $\langle A|B\rangle$  equals 0 even when neither  $|A\rangle$  nor  $|B\rangle$  equal zero.
  - ▶ In such cases  $|A\rangle$  and  $|B\rangle$  are orthogonal (perpendicular).
  - ▶ We can also see this in geometric representation...
- ▶ In geometrical representation:
  - ▶  $\langle A|B\rangle$  equals the length of  $|A\rangle$  times the length of  $|B\rangle$  times the cosine of the angle between  $|A\rangle$  and  $|B\rangle$ .
  - ▶  $\langle A|B\rangle = |A| \times |B| \times \cos\theta$
- ▶ The cosine of 90 degrees is 0.
  - ▶ So if neither  $|A\rangle$  nor  $|B\rangle$  are 0 yet  $\langle A|B\rangle$  equals 0 it must be because  $|A\rangle$  and  $|B\rangle$  are orthogonal.

# Orthogonality

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- ▶ We can determine whether vectors  $|A\rangle$  and  $|B\rangle$  are orthogonal by determining whether  $\langle A|B\rangle = 0$ .
- ▶ Orthogonal vectors are important because they constitute coordinate systems or *bases*.
- ▶ An important class of bases are called *orthonormal bases*.

# Orthonormal bases

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- ▶ Orthonormal bases are useful coordinate systems.
  - ▶ An orthonormal basis of an N-dimensional space is a collection of N mutually orthogonal vectors (in that space) of length 1.
- ▶ Ortho = orthogonal.
- ▶ Normal = norm-1 (i.e. length = 1).
- ▶ “Basis” because all vectors in the space can be defined in terms of such vectors.

# Importance of orthonormal bases

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- ▶ We saw earlier that any vector can be written as a (weighted) sum of other vectors.
- ▶ Any vector in a given  $N$ -dimensional space can be written as a (weighted) sum of *basis* vectors.
- ▶ So a small set of ( $N$ ) length-1 vectors can be used to define all vectors in the  $N$ -dimensional space.

# Illustration of orthonormal basis

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- ▶ Let  $|A\rangle$  and  $|B\rangle$  form an orthonormal basis of a 2D vector space.
- ▶ Vector  $|C\rangle$  in that space can then be defined:

$$|C\rangle = c_1|A\rangle + c_2|B\rangle$$

- ▶ Where the expansion coefficients are:

$$c_1 = \langle C|A\rangle$$

$$c_2 = \langle C|B\rangle$$



# Operators

# Operators

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- ▶ Operators “operate on” vectors to create new vectors.
- ▶ Applying operator  $O$  to vector  $|B\rangle$  is written  $O|B\rangle$ .
- ▶  $O|B\rangle = |B'\rangle$  for any vector  $|B\rangle$  in the vector space on which  $O$  is an operator.

# Examples of operators

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- ▶ Operators can:
  - ▶ Transform the direction of a vector (i.e. can rotate a vector).
    - ▶ E.g. The reflection operator, which gives the reflection of the vector on the other side of a given line.
  - ▶ Transform the length of a vector (i.e. multiply a vector by some value).
    - ▶ E.g. The halving operator, which has the effect of multiplying the vector by 0.5.
  - ▶ Transform both direction and length.
    - ▶ E.g. the x-projection operator, which projects a vector onto the x-axis by transforming  $\begin{bmatrix} x \\ y \end{bmatrix}$  into  $\begin{bmatrix} x \\ 0 \end{bmatrix}$
  - ▶ Do nothing
    - ▶ The unit operator that has the effect of multiplying the vector by 1.

# Linear operators

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- ▶ The operators that concern us are linear operators.
  - ▶  $O(|A\rangle + |B\rangle) = O|A\rangle + O|B\rangle$ 
    - ▶ The result of applying  $O$  to the sum of  $|A\rangle$  and  $|B\rangle$  is the same as the sum of the result of applying  $O$  to  $|A\rangle$  and the result of applying  $O$  to  $|B\rangle$ .
  - ▶  $O(c|A\rangle) = c(O|A\rangle)$ 
    - ▶ The result of applying  $O$  to the vector obtained by multiplying  $|A\rangle$  by  $c$  is the same as the product of  $c$  and the result of applying  $O$  to  $|A\rangle$ .

# Representing operators

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- ▶ (Linear) Operators are represented arithmetically as *matrices*.
- ▶ An operator on an N-dimensional space is represented by a matrix of  $N^2$  numbers.

Operator on a 2D space:

$$\begin{bmatrix} 0 & 1 \\ 2 & 6 \end{bmatrix}$$

Operator on a 3D space:

$$\begin{bmatrix} 3 & 4 & 5 \\ 12 & -9 & 0 \\ 0 & 1 & 5 \end{bmatrix}$$

# Multiplying operators by vectors

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- ▶ To multiply an operator by a vector:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

- ▶ To memorise this:
  - ▶ Think of taking the top line (a b) of the matrix, rotating it so that it matches up with the vector (multiplying a by x and b by y), and then adding  $ax$  to  $by$  to get the top entry for the resulting vector. The bottom entry is obtained by doing the same with the bottom line of the matrix.



# Eigenvectors

# Eigenvectors

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- ▶ In some cases we find the following result:
  - ▶  $O|B\rangle = \#|B\rangle$  for some number  $\#$ .
  - ▶ I.e. the operator returns the original vector multiplied by some number.
- ▶ In such cases we say that  $|B\rangle$  is an eigenvector of  $O$  with eigenvalue  $\#$ .
  
- ▶ Important because:
  - ▶ The only possible results that can be obtained from the measurement of a property that can be represented by the operator  $O$  will be the eigenvalues of  $O$ .



# The five principles of quantum mechanics



# The five principles

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- ▶ **Physical states**
  - ▶ Represented by certain kinds of vectors.
- ▶ **Measureable properties**
  - ▶ Represented by certain kinds of operators.
- ▶ **Dynamics**
  - ▶ Schrödinger equation changes vector direction over time, deterministically and linearly.
- ▶ **The connection with experiment**
  - ▶ The eigenstate-eigenvalue link: a state possesses the value  $v$  of a property represented by operator  $O$  if and only if that state is an eigenstate of  $O$  with eigenvalue  $v$ .
- ▶ **Collapse**
  - ▶ Measurement of the property represented by operator  $O$  when the measured system is not in an eigenstate of  $O$  will collapse the system into such an eigenstate, with a certain probability.

# Physical states

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- ▶ Physical systems are associated with vector spaces.
- ▶ Each length-1 vector in a physical system's vector space represents a possible physical state of the system.
  - ▶ Not *exactly* because  $|A\rangle$  and  $-|A\rangle$  will represent the same physical state.
  - ▶ Length-1 because probabilities must add to 1.
- ▶ Useful for representing superposition and property incompatibility.
  - ▶ Superpositions as weighted sums where the weights concern probabilities.
  - ▶ Allows superposition of one property to be equivalent to a definite instantiation of another (incompatible) property.

# Measurable properties

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- ▶ Physical systems (again) are associated with vector spaces.
- ▶ Measurable properties of physical systems are represented by linear operators on the vector spaces associated with those physical systems.
- ▶ The representation relation (between operators and properties) is called *the eigenstate-eigenvalue link*.
  - ▶ Note: Albert does not use this name (he doesn't offer any name) but it has since caught on in the literature.

# Measurable properties

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- ▶ **The eigenstate-eigenvalue link**
  - ▶ A state possesses the value  $v$  of a property represented by operator  $O$  if and only if that state is an eigenstate of  $O$  with eigenvalue  $v$ .
- ▶ **Important principle!**
  - ▶ It's a translation rule that translates physical notions (state, property, value of property) into mathematical objects (eigenvector, operator, eigenvalue).
- ▶ **Example:**
  - ▶ If the state of  $S$  is represented by an eigenvector of the colour operator, with eigenvalue  $e$  (say, black), then  $S$  is black.
  - ▶ Let's take this a bit slower...

# Measurable properties

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- ▶ Let's construct a 2D vector space where we can represent hardness.
- ▶ Constraint: they must be length-1 vectors.
  - ▶ Then the simplest vectors will be:

$$|hard\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |soft\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- ▶ Since  $\langle hard|soft\rangle = 0$  these vectors are orthogonal.
- ▶ They therefore form an orthonormal basis for the 2D space.

# Measurable properties

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- ▶ We can now work out what the hardness operator (H) must be.
- ▶ Constraint: It must be such that  $H|hard\rangle$  yields an eigenvector and  $H|soft\rangle$  yields an eigenvector (with distinct eigenvalues).
- ▶ So let the hardness operator be: 
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
- ▶ Let's see if that works...

# Measurable properties

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- ▶ Vectors stipulated to represent hard and soft:

$$|hard\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |soft\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- ▶ Suggested matrix for hardness:  $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

- ▶ Using the formula for operator/vector multiplication:

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

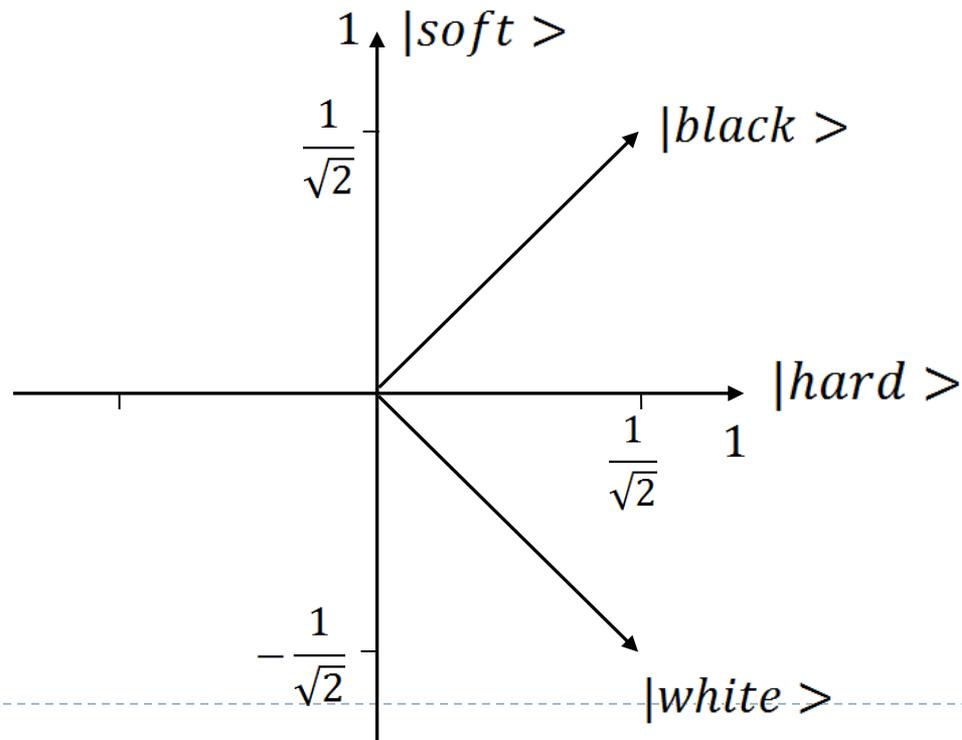
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = -1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- ▶ Hard is an eigenstate with eigenvalue +1 while soft is an eigenstate with eigenvalue -1.
  - ▶ So far so good!

# Measurable properties

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- ▶ A further constraint: our formalism must (somehow) represent states of hardness as superpositions of colour states (and vice versa).
- ▶ We can do this by associating black and white with vectors in the vector space we have just constructed:



# Measurable properties

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- ▶ In arithmetical form:

$$|black\rangle = \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad |white\rangle = \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

- ▶ We can now state the incompatibility relations as follows:

$$|black\rangle = \frac{1}{\sqrt{2}}|hard\rangle + \frac{1}{\sqrt{2}}|soft\rangle$$

$$|white\rangle = \frac{1}{\sqrt{2}}|hard\rangle - \frac{1}{\sqrt{2}}|soft\rangle$$

$$|hard\rangle = \frac{1}{\sqrt{2}}|black\rangle + \frac{1}{\sqrt{2}}|white\rangle$$

$$|soft\rangle = \frac{1}{\sqrt{2}}|black\rangle - \frac{1}{\sqrt{2}}|white\rangle$$

# Measurable properties

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- ▶ Now we just need to make sure that we can define a colour operator.

- ▶ The matrix in these equations will enable us to treat +1 as the eigenvalue for black and -1 as the eigenvalue for white.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = 1 \begin{bmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ -\frac{1}{\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \\ 1 \end{bmatrix} = -1 \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ -\frac{1}{\sqrt{2}} \\ 1 \end{bmatrix}$$

# Measurable properties

---

- ▶ Recall the eigenstate-eigenvalue link
  - ▶ A state possesses the value  $v$  of a property represented by operator  $O$  if and only if that state is an eigenstate of  $O$  with eigenvalue  $v$ .
- ▶ Hopefully this rule is now somewhat more clear.
  - ▶ Please study it, it's important for this course!
- ▶ The homework exercises will help – please complete them before Tuesday (10/2)!

# Dynamics

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- ▶ There is a law of nature that determines changes in a physical system's state over time given that system's interactions, described by the Schrödinger equation.
- ▶ Here's what's important about the equation for our purposes:
  - ▶ It changes the direction (not the length) of state vectors.
    - ▶ State vectors must always be length 1.
  - ▶ It is deterministic.
    - ▶ Discussed in lecture 1.
  - ▶ It is linear.
    - ▶ This one is very important...

# Dynamics

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- ▶ Linearity...
- ▶ If the law guarantees the following:
  - ▶ If a physical system is in state  $|A\rangle$  then it will evolve into state  $|A'\rangle$ .
  - ▶ If a physical system is in state  $|B\rangle$  then it will evolve into state  $|B'\rangle$ .
- ▶ Then the law also guarantees that:
  - ▶ If a physical system is in the superposition state  $\#|A\rangle + \#|B\rangle$  then it will evolve into the superposition state  $\#|A'\rangle + \#|B'\rangle$ .
- ▶ Linearity plays a crucial role in Albert's formulation of the measurement problem (chapter 4).

# The connection with experiment

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- ▶ Recall (again) the eigenstate-eigenvalue link:
  - ▶ A state possesses the value  $v$  of a property represented by operator  $O$  if and only if that state is an eigenstate of  $O$  with eigenvalue  $v$ .
- ▶ So a measurement of the property represented by  $O$ , on a system in state  $v$ , will yield result  $v$ .
- ▶ What about the same measurement (of the property represented by  $O$ ) on a system that is not in an eigenstate of  $O$ ? What result will it yield?

# The connection with experiment

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- ▶ Let  $|B=b_i\rangle$  denote the eigenvector of property B with eigenvalue  $b_i$ .
  - ▶ Thus  $|\text{Colour}=-1\rangle$  denotes  $|\text{white}\rangle$ .
- ▶ If  $|A\rangle$  is the state vector of a system then the probability that a B measurement gives  $|B=b_i\rangle$  is:

$$(\langle a|B = b_i \rangle)^2$$

- ▶ Let's illustrate with some cases...

# The connection with experiment

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- ▶ Simplest cases:

- ▶ What is the probability that a hardness measurement on a soft electron will yield *soft*?

$$\begin{aligned} & (\langle a|B = bi \rangle)^2 \\ &= (\langle soft|soft \rangle)^2 \\ &= \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)^2 \\ &= (0 \times 0 + 1 \times 1)^2 \\ &= 1 \end{aligned}$$

# The connection with experiment

---

▶ Simplest cases:

- ▶ What is the probability that a hardness measurement on a soft electron will yield *hard*?

$$\begin{aligned} & (\langle a|B = bi \rangle)^2 \\ &= (\langle soft|hard \rangle)^2 \\ &= \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)^2 \\ &= (0 \times 1 + 1 \times 0)^2 \\ &= 0 \end{aligned}$$

# The connection with experiment

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▶ Interesting cases:

- ▶ What is the probability that a hardness measurement on a *white* electron will yield hard?

$$\begin{aligned} & (\langle a|B = bi \rangle)^2 \\ = & (\langle white|hard \rangle)^2 & = \left( \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)^2 \\ & & = \left( \frac{1}{\sqrt{2}} \times 1 + -\frac{1}{\sqrt{2}} \times 0 \right)^2 \\ & & = \left( \frac{1}{\sqrt{2}} \right)^2 \\ & & = 0.5 \end{aligned}$$

# The connection with experiment

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- ▶ This last result captures the fact that hardness measurement on a white electron gives 50/50 results.
- ▶ This is how quantum mechanics predicts the statistical results of experiments.

- ▶ Recall the incompatibility relation:

$$|white\rangle = \frac{1}{\sqrt{2}}|hard\rangle - \frac{1}{\sqrt{2}}|soft\rangle$$

- ▶  $|white\rangle$  is equivalent to a weighted sum of hardness states.
  - ▶ The weights relate to the probability of measurement outcomes through:  $(\langle a|B = bi\rangle)^2$

# Collapse

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- ▶ Continuing with the example; a hardness measurement on a white electron yields a hard result with probability 0.5 and a soft result with probability 0.5. Assume a hard result. Then the electron has evolved from:

$$\frac{1}{\sqrt{2}}|hard\rangle - \frac{1}{\sqrt{2}}|soft\rangle$$

- ▶ To:

$$|hard\rangle$$

- ▶ Measurement appears to have collapsed the superposition state into a definite hardness state.

# The five principles of quantum mechanics

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- ▶ **Physical states**
  - ▶ Represented by certain kinds of vectors.
- ▶ **Measureable properties**
  - ▶ Represented by certain kinds of operators.
- ▶ **Dynamics**
  - ▶ Schrödinger equation changes vector direction over time, deterministically and linearly.
- ▶ **The connection with experiment**
  - ▶ The eigenstate-eigenvalue link: a state possesses the value  $v$  of a property represented by operator  $O$  if and only if that state is an eigenstate of  $O$  with eigenvalue  $v$ .
- ▶ **Collapse**
  - ▶ Measurement of the property represented by operator  $O$  when the measured system is not in an eigenstate of  $O$  will collapse the system into such an eigenstate, with a certain probability.

# Applying quantum mechanics

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- ▶ How do we predict the behaviour of a physical system using these five principles?
- ▶ General approach:
  - ▶ [1] Identify the vector space associated with that system i.e. find a suitable basis set of vectors.
  - ▶ [2] Identify the operators associated with the system's properties i.e. calculate eigenvectors and eigenvalues of appropriate operators.
  - ▶ [3] Map out correspondences between physical states and individual vectors i.e. decide how to label the state vectors.
  - ▶ [4] Ascertain the present state vector of the system by means of measurement i.e. determine initial labels...

# Applying quantum mechanics

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- ▶ **General approach continued:**

- ▶ [5] The time evolution of the system is then determined by the dynamics i.e. the Schrödinger equation.

- ▶ [6] The probabilities of particular outcomes of a measurement carried out at a future time can then be calculated via:

$$(\langle a|B = bi \rangle)^2$$

- ▶ [7] The effects of measurement are taken into account using the collapse postulate – the state collapses to an appropriate eigenstate of the measured property.

- ▶ [8] Then [5] – [7] are just repeated over again.

# The measurement problem

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- ▶ Recall our original formulation of the measurement problem: quantum mechanics postulates a deterministic law and an indeterministic law, but they can't both obtain at once.
- ▶ We can now see how this arises in the formalism:
  - ▶ Most of the time the state vector evolves (changes direction) deterministically.
  - ▶ Sometimes it evolves indeterministically: when we measure the property of a system whose state vector is not an eigenvector of the operator associated with that property.

# The Copenhagen interpretation

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- ▶ The Copenhagen interpretation is an influential interpretation of quantum mechanics that comes from Neils Bohr.
- ▶ Albert describes it as follows:
  - ▶ “The right way to think about superpositions of, say, being black and being white is to think of them as situations wherein color talk is unintelligible. Talking and enquiring about the color of an electron in such circumstances is (on this view) like talking or inquiring about, say, whether the number 5 is still a bachelor. On this view the contradictions of chapter 1 go away because superpositions are situations wherein the superposed predicates just don’t apply.” (p38.)
- ▶ Measurement therefore brings the world into a state in which we can meaningfully talk about it.

# Tuesday's lecture...

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- ▶ Other spin space properties
- ▶ Commutators
- ▶ Coordinate space and position
- ▶ Composite systems
  
- ▶ We will then be in a position to apply quantum mechanics to the experiments and begin to discuss the measurement problem more carefully.
  
- ▶ For now let's digress into some philosophy of mind...



# The mind-body problem



# The mind-body problem

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- ▶ The hard problem of consciousness
  - ▶ How do collections of physical processes (like neuron firings) give rise to subjective conscious experience?
    - ▶ Will discuss in detail.
- ▶ The problem of free will
  - ▶ How can our actions be free (so that we are ultimately responsible for them) when those actions are physical events determined by prior physical events and physical laws?
    - ▶ Will discuss briefly.
- ▶ The problem of intentionality
  - ▶ How can a thought or an experience be *about* some other state of the world?
    - ▶ Perhaps not relevant for our purposes?
- ▶ The problem of the self
  - ▶ What is the self, how is it related to the brain, how is personal identity over time possible? Etc.
    - ▶ Perhaps not relevant either, but consider what personal identity amounts to according to many-worlds theory!

# The hard problem of consciousness

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- ▶ Refers to: our (present) inability to explain conscious experience in terms of underlying physical states (e.g. brain states).
- ▶ Many correlations between brain states and conscious states have been established (by fMRI-scans etc.), but a correlation is not an explanation. A correlation is something to be explained.
- ▶ Real life manifestation: we don't know whether insects or fish or robots are conscious. Also: problem of other minds.

# Easy problems versus the hard problem

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- ▶ Chalmers (1995) distinguishes the easy problems from the hard problem. (See also: slide 2)
- ▶ Easy problems: explaining cognitive functions:
  - ▶ Reportability of mental states, memorising of mental states etc.
    - ▶ Definable in terms of functional roles.
    - ▶ Explained by locating underlying mechanism that plays these roles.
- ▶ Hard problem: explaining conscious experience:
  - ▶ The “what-it-is-like-ness” of experience: the painfulness of pain, the redness of red, the various qualities of experience etc.
    - ▶ Not definable in terms of functional roles.
    - ▶ So not explainable in the ordinary manner.
    - ▶ Given a information about the physical realiser of cognitive functions the question will remain: but why should it be *conscious*?

# Physicalism versus dualism

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## ▶ Physicalism:

- ▶ All mental states (including states of consciousness) are identical to physical states (e.g. brain states).
  - ▶ Must account for the hard problem in some way.

## ▶ Dualism:

- ▶ States of consciousness are not identical to physical states. Experiences are distinct but may be related to physical states by further laws of nature (psychophysical laws).
  - ▶ Does not face the hard problem!
  - ▶ But does face the interaction problem: what are these psychophysical laws?

# Why be a physicalist?

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- ▶ The causal closure argument (or a standard version of it):
  - ▶ **(i) Causal closure:** modern physics demonstrates that non-physical minds cannot causally affect physical systems, because all physical effects are accounted for by physical causes;
  - ▶ **(ii)** consciousness causally affects some physical systems (brains and bodies);
  - ▶ **(iii)** therefore, consciousness must (somehow!) be physical.
- ▶ This is the standard argument for physicalism.
- ▶ Why accept (i) causal closure?

# Daniel Dennett's defence of physicalism

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- ▶ “[Experiences according to dualism] ex hypothesi, are not physical; they are not light waves or sound waves or cosmic rays or streams of subatomic particles. No physical energy or mass is associated with them. How, then, do they get to make a difference to what happens in the brain cells they must affect, if the mind is to have any influence over the body? **A fundamental principle of physics** is that any change in the trajectory of any physical entity is an acceleration requiring the expenditure of energy, and where is this energy to come from? **It is this principle of the conservation of energy** that accounts for the physical impossibility of "perpetual motion machines," and the same principle is apparently violated by dualism. This confrontation between **quite standard physics** and dualism has been endlessly discussed since Descartes's own day, and **is widely regarded as the inescapable and fatal flaw of dualism.**
- ▶ Daniel Dennett: *Consciousness Explained* (1991); p35.

# Quite standard physics?

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- ▶ **Cosmologist Sean Carroll writes:**

- ▶ “It’s clear that cosmologists have not done a very good job of spreading the word about something that’s been well-understood since at least the 1920’s: energy is not conserved in general relativity.”

- ▶ “When the space through which particles move is changing, the total energy of those particles is not conserved.”

- ▶ <http://www.preposterousuniverse.com/blog/2010/02/22/energy-is-not-conserved/>

- ▶ **Energy conservation also fails in textbook quantum mechanics!**

- ▶ Collapse typically violates energy conservation, as we shall see in forthcoming lectures.

# David Papineau's defence of physicalism

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- ▶ “the case against interactionist dualism hinges crucially on the empirical thesis that all physical effects already have physical causes. It is specifically this claim that makes it difficult to see how dualist states can make a causal difference to the physical world.”
  - ▶ <http://plato.stanford.edu/archives/spr2009/entries/naturalism/#MenProCauCloArg>
- ▶ Unlike Dennett's argument, Papineau's is not inconsistent with general relativity.
  - ▶ Violations of energy-conservation are *caused* by space-time distortions i.e. by *physical* causes.
- ▶ But are all known *quantum* effects accounted for by physical causes?

# Papineau on quantum mechanics

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- ▶ “Sometimes it is suggested that the indeterminism of modern quantum mechanics creates room for sui generis non-physical causes to influence the physical world. However, even if quantum mechanics implies that some physical effects are themselves undetermined, it provides no reason to doubt a quantum version of the causal closure thesis, to the effect that the chances of those effects are fully fixed by prior physical circumstances. And this alone is enough to rule out sui generis non-physical causes. For such sui generis causes, if they are to be genuinely efficacious, must presumably make an independent difference to the chances of physical effects, and this in itself would be inconsistent with the quantum causal closure claim that such chances are already fixed by prior physical circumstances. Once more, it seems that anything that makes a difference to the physical realm must itself be physical.”
  - ▶ Is this a good argument?

# Consciousness and collapse

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- ▶ Perhaps the chances of measurement outcomes accounted for by physics (via the born rule).
- ▶ But what about collapse itself?
- ▶ Do we have reason to rule out the hypothesis that consciousness causes collapse?

# QM and psychophysical principles

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- ▶ Physicalist psychophysical physical principles
  - ▶ Conscious state C is *identical to* physical (brain) state P.
- ▶ Dualist psychophysical physical principles
  - ▶ Conscious state C is *lawfully correlated with* physical (brain) state P.
- ▶ Will solving the measurement problem require appeal to psychophysical principles?
  - ▶ E.g. Monton's (physicalist!) solution to the tails problem.
    - ▶ See: section 4.4 (p15) of my Four Tails Problem For Dynamical Collapse Theories.

# The problem of free will

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## ▶ Determinism

- ▶ All physical events are determined by initial conditions and deterministic laws. So we have no free will.

## ▶ Libertarianism

- ▶ We do have free will, so not all physical events are determined by initial conditions and deterministic laws

## ▶ Compatibilism

- ▶ All physical events are determined by initial conditions and deterministic laws. But we nonetheless have free will.
  - ▶ (rejects “incompatibilism”, which the first two assume)

## ▶ Determinism is inconsistent with textbook quantum mechanics.

- ▶ Does this affect the debate?
- ▶ Indeterministic choices?

# The problem of free will

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- ▶ What about the consciousness causes collapse hypothesis?
  - ▶ On the face of it: no help since consciousness triggers events that collapse probabilistically.
    - ▶ Probabilities are “determined” by the Born rule, not by the “self” (perhaps this is what Papineau was getting at?)
  - ▶ But: room for control over consciousness and collapse?
  - ▶ Perhaps intelligent concentration violates the Born rule in some way?
    - ▶ Some have argued for this!
      - Schwartz, J.M., Stapp, H.P., and Beauregard, M. 2005. Quantum theory in Neuroscience and Psychology: a Neurophysical Model of Mind/Brain Interaction. *Philosophical Transactions of the Royal Society B* 360, 1309–1327.
      - Radin, D et. al. 2012. Consciousness and the double-slit interference pattern: Six experiments. *Physics Essays* 25, 2.
    - ▶ Note: this is highly speculative and controversial stuff!