Philosophy of Modern Physics

PHIL3622 Lecture 12: 24/10/2013
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Today's lecture

- The basic idea behind Bohmian mechanics
- Laws for a single structureless particle.
- Generalising the laws
- Experiments
- Nonlocality
- Explaining the appearance of collapse
- Objections

Bohmian mechanics: basic idea

The problem of outcomes

- The following three claims are mutually inconsistent.

  - **A. The wave-function of a system is complete i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.**
    - Bohmian mechanics denies proposition A.

  - B. The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger equation).

  - C. Measurements always (or at least usually) have determinate outcomes, i.e., at the end of the measurement the device indicates a definite physical state.

Denying proposition A: what's needed

- Precisely specify the additional ontology and what laws govern that ontology.

- Be consistent with Bell's result (that there is no local realist additional variables theory).

- Explain how the additional variables determine definite measurement outcomes.

- Explain the appearance of indeterministic collapse.

Bohmian mechanics: the basic idea

- The formalism is largely the same as standard quantum theory but the metaphysics is very different.

- A particle's wave-function is *not a mathematical description* of that particle's physical state.
- Rather, the wave-function is *a physical thing* like a force field that pushes the particle around.
  - Its physical properties are *amplitudes* assigned to points in space.

- Particles always have determinate positions – no superpositions.
- The theory is about describing the deterministic trajectories of these particles.

Bohmian mechanics: the basic idea

- The wave-function (the physical object) evolves in accordance with the deterministic linear Schrödinger equation.

- The particles evolve in accordance with the deterministic velocity function (sometimes: “Bohm’s guidance equation”).
  - This is a function of the initial positions of the particles and the wave-function.
  - The wave-function causally affects the particles but the particles do not causally affect the wave-function.

Bohmian mechanics: the basic idea

- The theory is completely deterministic.
  - So there are no objective probabilities.
    - Nor any need to reconsider our concept of probability in order to account for probability (as in Many Worlds theory).
  - There are only subjective probabilities.
    - i.e. measures of our ignorance about the physically determined future.

- Bohm’s theory is able to show that quantum probabilities emerge from our inescapable ignorance of the positions of particles.

Formalism for structureless particle

Velocity function

- Consider a structureless particle p confined to one spatial dimension.
- p's velocity is given by the value of the *velocity function*: V(x).
- V(x) is determined (at a time) by p's wave-function (at that time) by means of an algorithm.
  - Albert denotes the algorithm $V[\psi(x)]$ where $\psi(x)$ is p's wave-function.
- Upshot: if you know p's position and you know p's wave-function then you can calculate p's velocity.

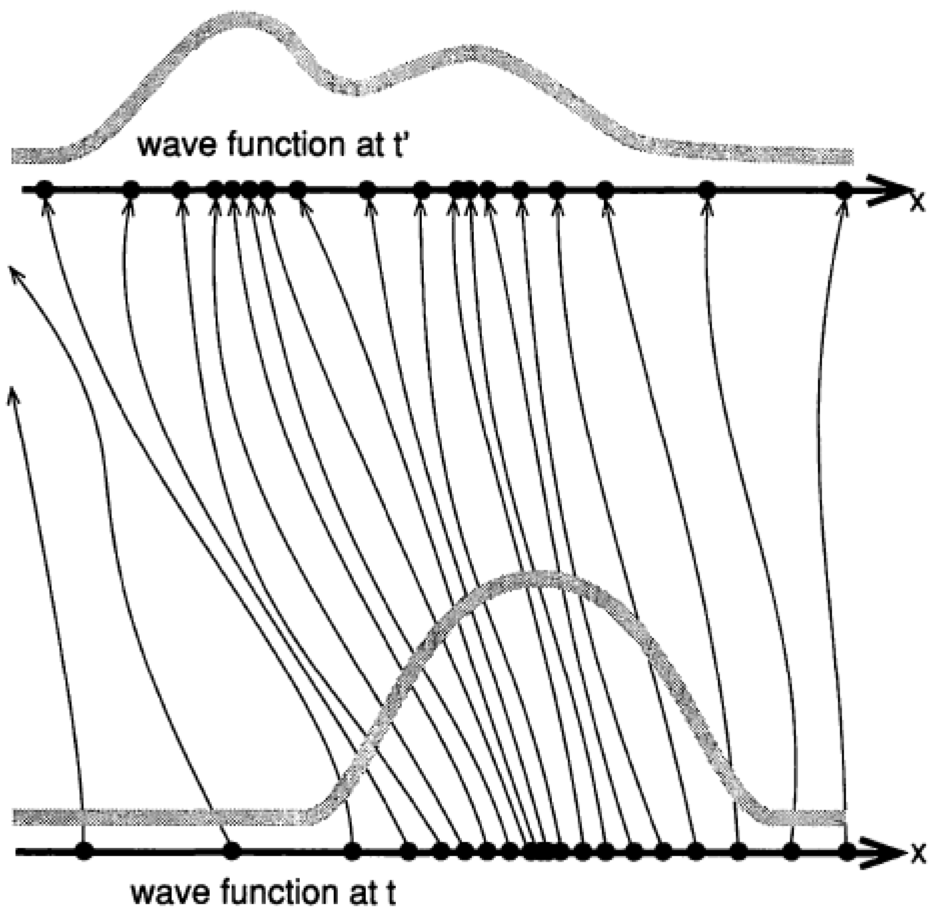
Velocity function

- The velocity function is sometimes called the “guidance equation”.
  - Where the particle moves to (in a given instant of time) depends on the shape of the wave-function: the wave-function guides particles around.

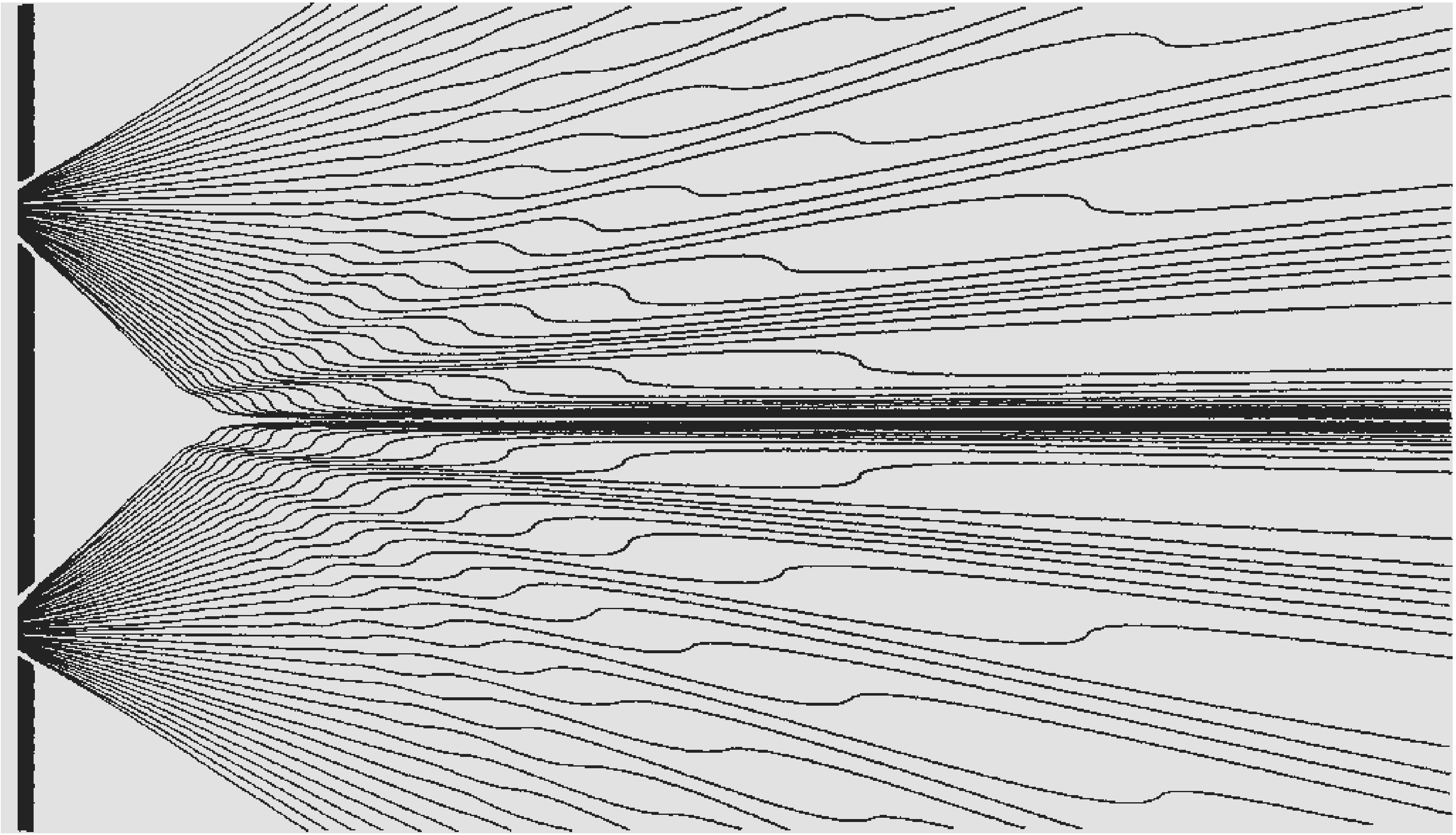
- If you know p’s velocity at time t then you know p’s position at the next instant t+1.
  - Now find the wave-function at t+1 in the standard way (Schrödinger equation).
  - Now reapply the velocity function and you’ve got p’s position at the next instant (t+2).
  - And so on...
    - So you can calculate deterministic trajectories.

Depiction of the "guidance"

- P's wave-function evolves from time t to t'.
- The *density* of possible positions along the x-axis (at a time) is everywhere equal to the absolute value squared of the wave-function (at that time).
- "The particle gets carried along with the flows of the amplitude like a cork floating on a river" (p138).



Possible 2-slit trajectories



The statistical postulate

- If you don't know:
  - The present position of the particle
- But you do know:
  - The present wave-function of the particle
- Then to calculate the future trajectory of the particle:
  - The square of (the absolute value of) the **present** wave-function at a position will give the probability for the particle being located at that position (at the **present** time).
  - The square of (the absolute value of) the **future** wave-function at a position will give the probability for the particle being located at that position (at the **future** time).
- Because (as we shall see) we cannot know the present position of any particle, Bohmian mechanics recovers the standard probabilistic predictions of QM.

Generalising the laws

Three spatial dimensions

- The wave function is now a function of (or attributes amplitudes to) points in three dimensions: $\psi(x,y,z)$.
- The algorithm for calculating the velocity function is now three algorithms for calculating three velocity functions.

$$V_x\{\psi(x,y,z)\} = V_x(x,y,z)$$

$$V_y\{\psi(x,y,z)\} = V_y(x,y,z)$$

$$V_z\{\psi(x,y,z)\} = V_z(x,y,z)$$

- Thus the velocity in the x-direction for a particle located at x,y,z is a function of the amplitude at x,y,z.
  - Similarly for the velocity in the y and z directions.

Spin

- Spin properties (colour/hardness) are mathematical properties *of wave functions*.
- They play no role determining velocities.
  - Velocities determined by coordinate-space part of the wave-function alone.
  - Consider the velocity functions for separable states:

- $V_i(x,y,z) = V_i\{|black\rangle|\psi(x,y,z)\rangle\} = V_i\{|\psi(x,y,z)\rangle\}$
- $V_i(x,y,z) = V_i\{|white\rangle|\psi(x,y,z)\rangle\} = V_i\{|\psi(x,y,z)\rangle\}$
- $V_i(x,y,z) = V_i\{|whatever\rangle|\psi(x,y,z)\rangle\} = V_i\{|\psi(x,y,z)\rangle\}$
    - i = x, y, z.

Nonseparable states

- What if the state is nonseparable between spin and coordinate space? E.g.

$$a|\text{black}\rangle|\psi(x,y,z)\rangle + b|\text{white}\rangle|J(x, y, z)\rangle$$

- Then each of the different coordinate-space wave-functions contribute, in accordance with their “weight”, to determine the velocity functions:

$$V_i(x,y,z) = |a|^2 V_i\{\psi(x,y,z)\} + |b|^2 V_i\{J(x,y,z)\} \qquad (i = x,y,z)$$

Multiple-particle systems

- How to deal with composites?

- Let $|\psi_{1,2}\rangle$ designate the state of a 2-particle composite.
- Let $|x_1,x_2\rangle$ designate the state where particle 1 is located at x_1, particle 2 at x_2.

- The 2-particle wave-function associated with $|\psi_{1,2}\rangle$ is given by a function of x_1 and x_2:

- $\psi(x_1,x_2) = \langle\psi_{1,2}|x_1,x_2\rangle$

The 3N-dimensional ψ-space

- The 2-particle wave-function is a function of position in a 6-dimensional space.
  - The first three are those of particle 1, the second of particle 2.
    - A point in this space represents positions for both particles.

- Bohm's laws (for two particles) are formulated as if it were a single particle being pushed around in a 6D space:
  - The velocity functions are functions of the position of the

$$V_i(x,y,z,x',y',z') = V_i\{\psi(x,y,z,x',y',z')\} \qquad (i = x,y,z,x',y',z')$$

Upshots

- The 2-particle system is pushed around in the 6D space by the flows of the QM probabilities.

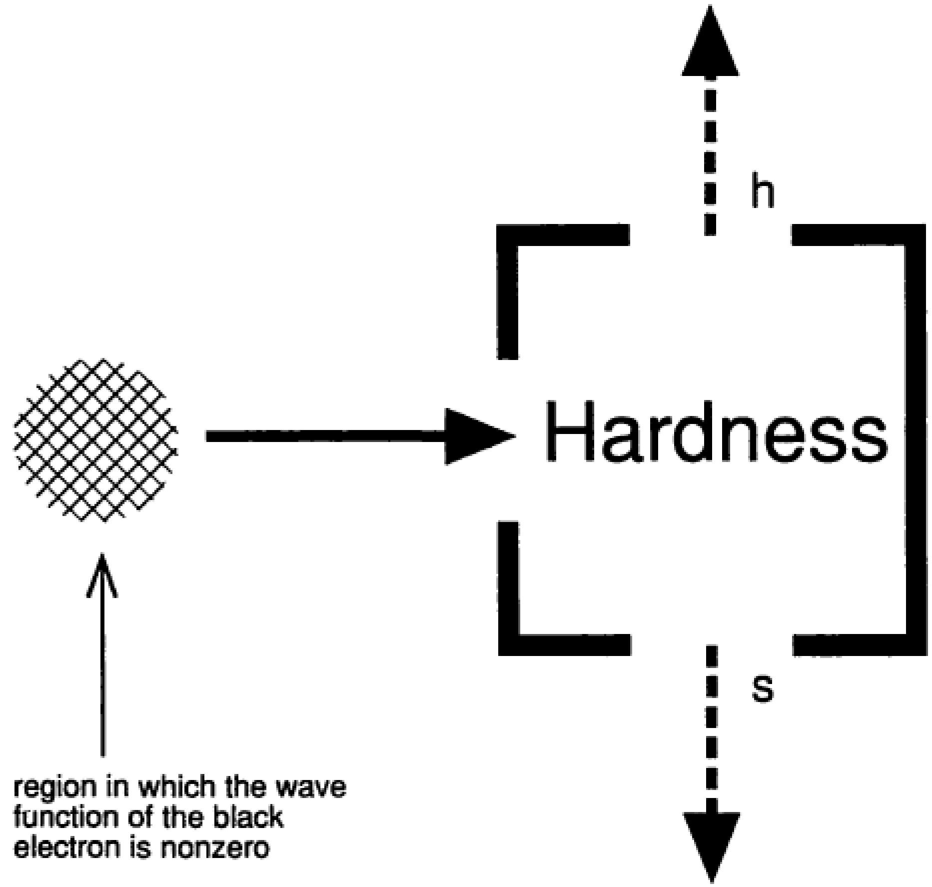
- The statistical postulate entails that if we don't know the positions of the two particles, only their wave-function, then we can predict the probabilities for their positions using Born's rule.

- So the initial positions of all particles in the universe and the particular shape of the wave-function are such that it's as if quantum mechanics is true.

Experiments

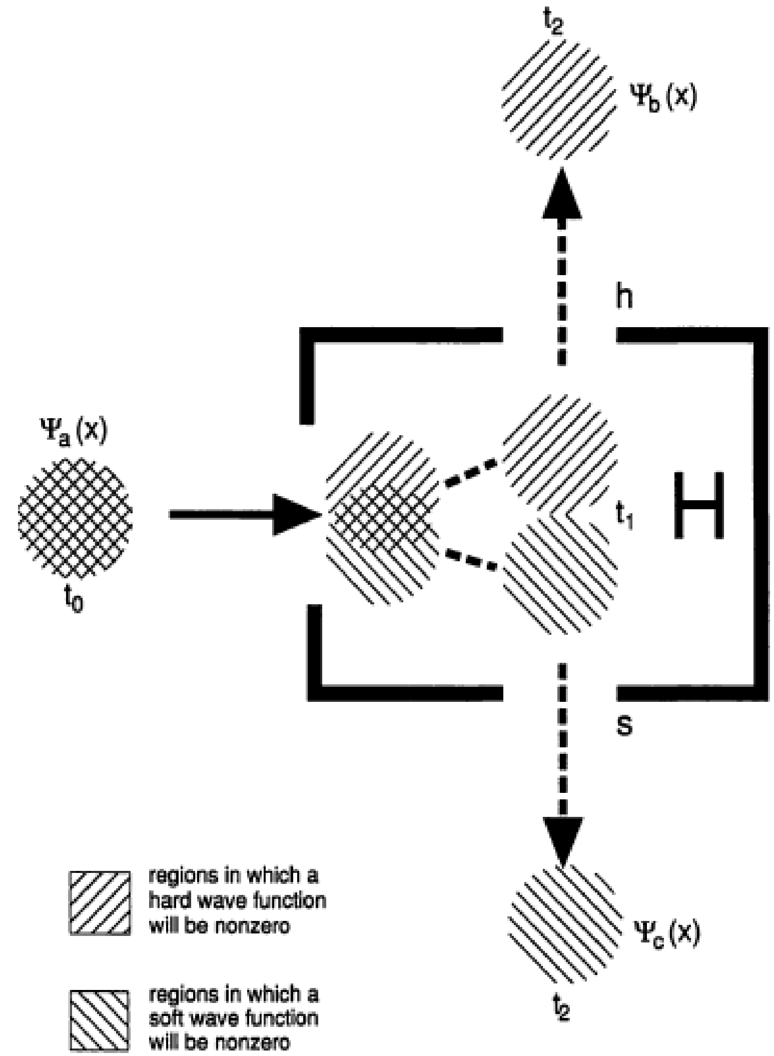
Spin boxes

- Consider an electron such that
  - (i) its wave-function is black.
  - (ii) it is situated in coordinate space as depicted.
- Let's look at how we can determine future electron position with certainty.



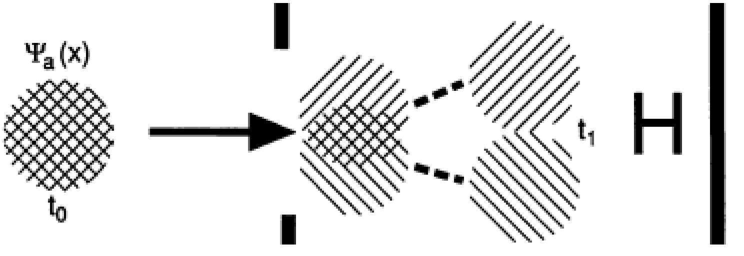
Spin boxes

- The hardness-dependent forces inside the box cause hardness eigenstates to evolve as depicted.
- Linearity then dictates:
  - |black›|ψ_a(x)›
  - →
  - 1/√2(|hard›|ψ_b(x)›+ |soft›|ψ_c(x)›)
    - |ψ_a(x)› = w-f nonzero only in a-vicinity and travelling right.
    - |ψ_b(x)› = w-f nonzero only in b-vicinity and travelling up.



Spin boxes

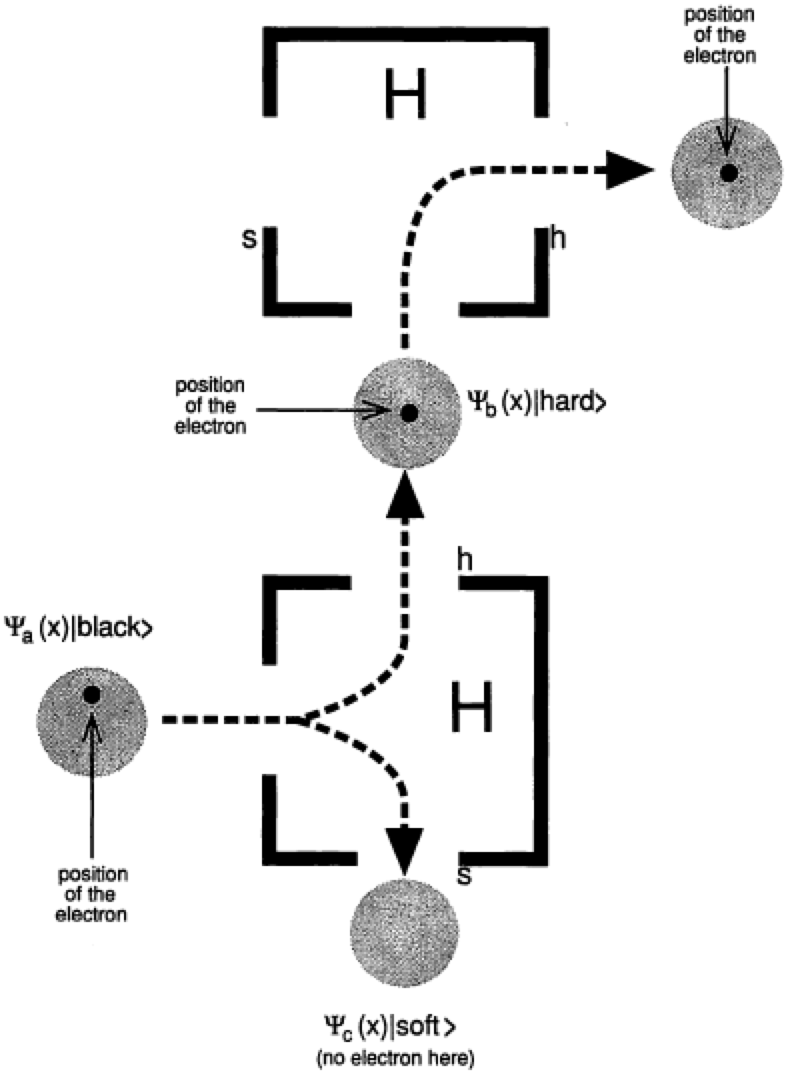
- Electron motion depends on its initial position and the "local currents" of the amplitudes.
- The superposition is equally weighted.
  - Hence if e is located in the intersection of the two branches its velocity in the vertical direction is zero.
  - And when the branches don't overlap e is located exclusively in one branch then it will follow that branch.



  - So if e starts out in the upper half it will exit the hard aperture, if the lower half, then the soft aperture.

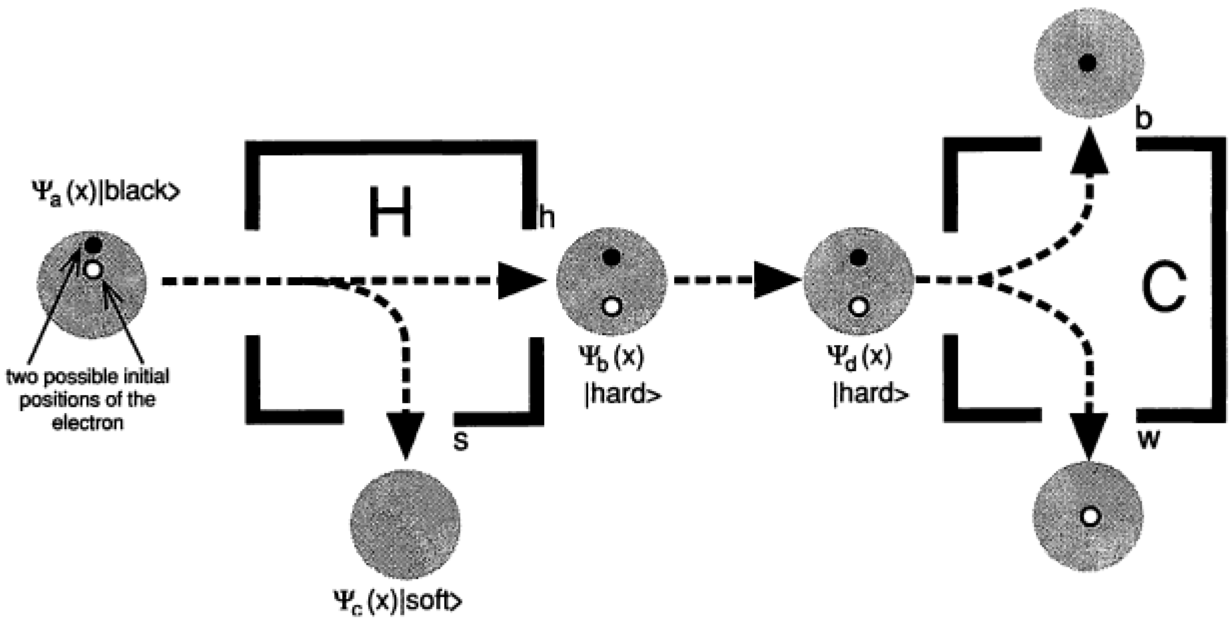
Spin boxes

- Let's say e starts out in the upper region, so exits the H-aperture.
- We then feed it into another hardness box.
- Since e is moving to regions where the amplitude of the soft part of its w-f is zero, the soft part has no effect on its motion.
  - So it gets carried along by the hard part.



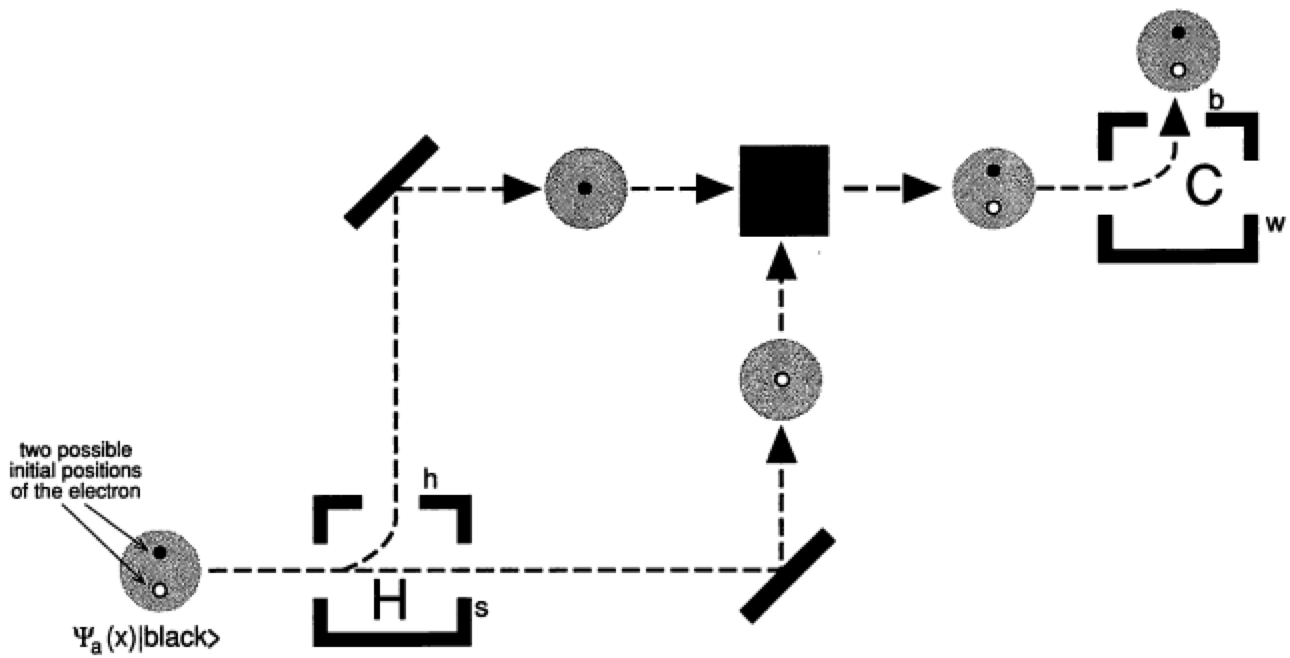
Spin boxes

- Now send e (initially located in the upper black region) through a hardness box then a colour box.
- Where e exits the colour box depends more precisely on its initial position:



2-path experiment

- What if the two w-f branches are reunited before entering the colour box?
- Then no matter its initial position within the initial a-region, e will be black.



- What if we insert a wall on one path?

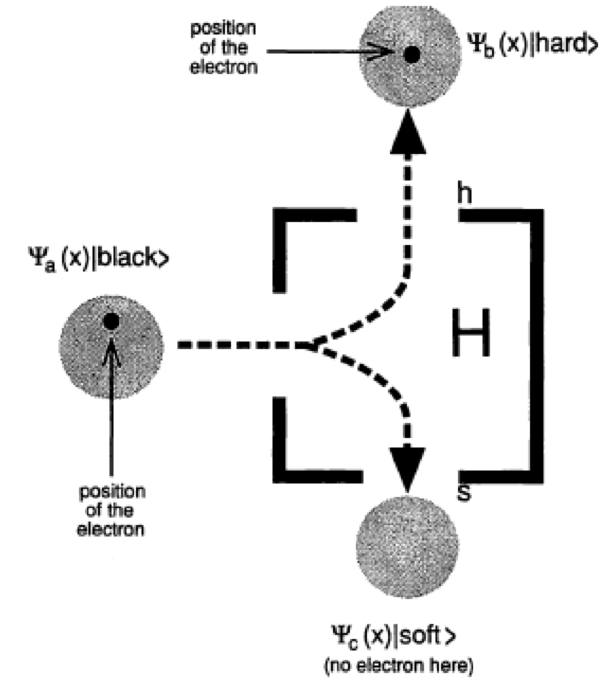
Upshot

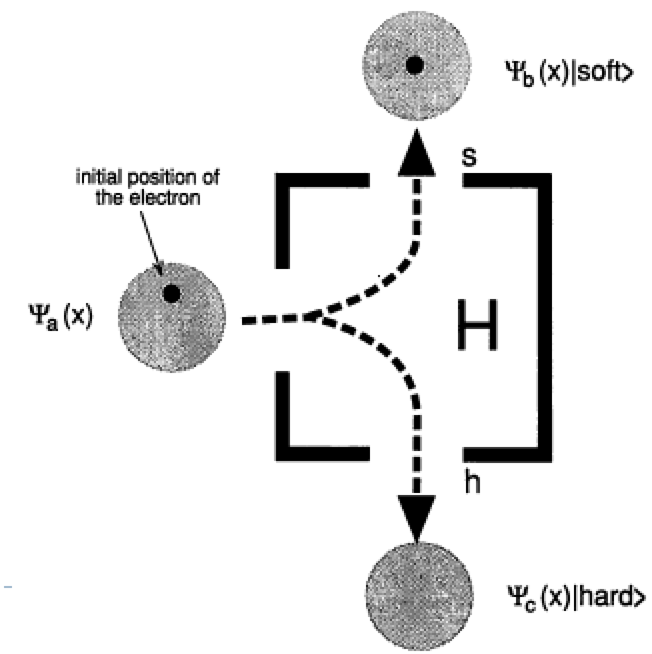
- If we don't know initial positions, only wave-functions, then the statistical postulate reproduces all the measurement outcomes (& their relative frequencies) predicted by QM.
  - After all: the experiments always come down to the final *positions* of the measured particles.

- So since Bohm gets everything right about positions he gets everything right.
  - As long as all measurement outcomes are recorded in the positions of things.
    - Compare GRW: measurement outcomes recorded in the *macroscopic* positions of things.

Contextuality

- Recall our first experiment where the electron was in the upper half, so exited the hard aperture.
- Now flip the hardness box.
- Rather than:
  - $|\text{black}\rangle|\psi_a(x)\rangle$
  - $\rightarrow$
  - $1/\sqrt{2}(|\text{hard}\rangle|\psi_b(x)\rangle + |\text{soft}\rangle|\psi_c(x)\rangle)$
- We get:
  - $|\text{black}\rangle|\psi_a(x)\rangle$
  - $\rightarrow$
  - $1/\sqrt{2}(|\text{soft}\rangle|\psi_b(x)\rangle + |\text{hard}\rangle|\psi_c(x)\rangle)$
- So electron exits soft aperture.





Contextuality

- So the outcome of a “hardness measurement” on an electron is not determined by electron’s position and wave-function.
  - But this is all there is the electron!
- The outcome depends on the context, even down to the orientation of the device.
  - So hardness is not an intrinsic property of electrons.
    - It’s a “contextual” property.
- *All* measureable properties other than position are contextual properties on this theory.
  - All spin-space properties, momentum, energy etc.
    - Theorems: Gleason 1957, Kochen and Specker 1967.

Nonlocality

Nonlocality

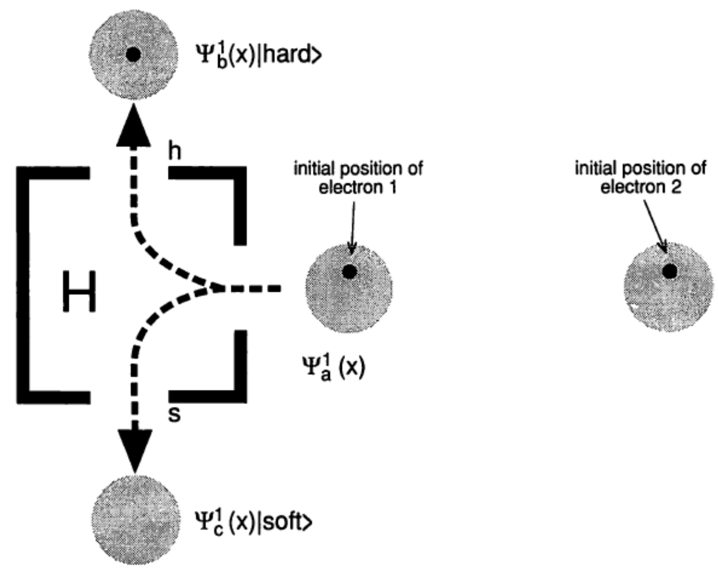
- If the composite wave-function is nonseparable then Bohm's laws are nonlocal.
  - 2-particle system: once the wave-function is fixed, the velocity in the x-direction of particle 1 at some moment will depend not only on *its* position but also on the position *of particle 2* at that moment (no matter how far apart they are).
    - Each of the 6 velocity functions will depend on the location of the entire 2-particle system in the 6D space.
    - The motions of p2 will change the location of the system in 6D space.

Nonlocality illustrated

- Two electrons with separable coordinate-space w-fs, but nonseparable spin:

$$1/\sqrt{2}\,(|\text{hard}\rangle_1|\psi_a(x)\rangle_1|\text{soft}\rangle_2|\psi_f(x)\rangle_2 \;-\; |\text{soft}\rangle_1|\psi_a(x)\rangle_1|\text{hard}\rangle_2|\psi_f(x)\rangle_2)$$

- Electron 1 starts in the upper region & passes through a "right-side-up" hardness box:



- It will end up in the b-region:

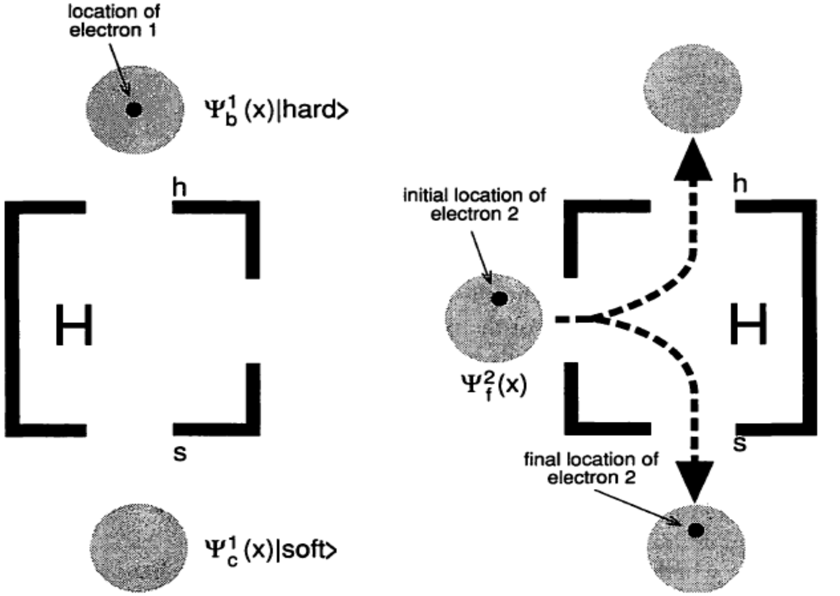
$$1/\sqrt{2}\,(|\text{hard}\rangle_1|\psi_b(x)\rangle_1|\text{soft}\rangle_2|\psi_f(x)\rangle_2 \;-\; |\text{soft}\rangle_1|\psi_c(x)\rangle_1|\text{hard}\rangle_2|\psi_f(x)\rangle_2)$$

Nonlocality illustrated

- But now the system is at a point in 6D space where the second term is zero.

$$1/\sqrt{2}\,(|\text{hard}\rangle_1|\psi_b(x)\rangle_1|\text{soft}\rangle_2|\psi_f(x)\rangle_2 \;-\; |\text{soft}\rangle_1|\psi_c(x)\rangle_1|\text{hard}\rangle_2|\psi_f(x)\rangle_2)$$

- So the second term has no effect on the system.
  - The system is "guided" only by the first term.
- It will therefore be *as if* they collapsed.
  - Electron 1 will behave as if it were hard.
  - Electron 2 will behave as if it were soft.
  - Independently of hardness box orientation or initial pos:



Nonlocality illustrated

- The trajectory of electron 1 through the hardness box brings about an "effective" collapse of the wave-function of the system.
  - Instantaneously, no matter how far apart.
  - "Effective collapse" means "it's as if it collapsed".

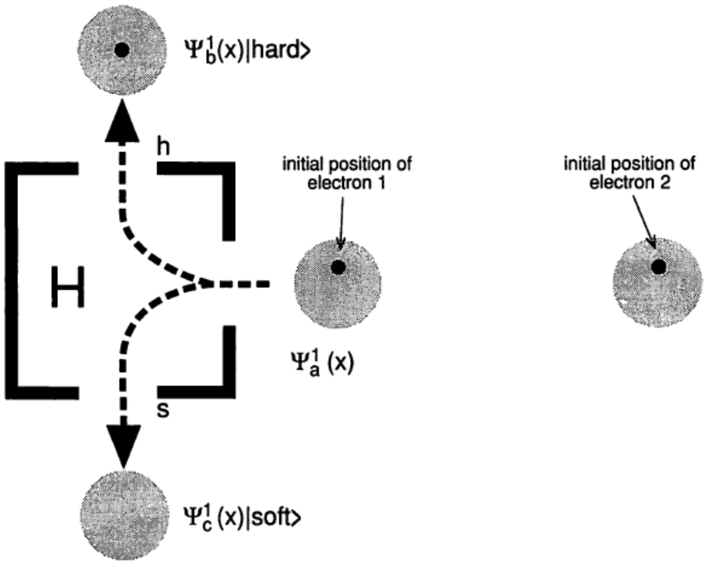
- And if all we know about the initial state is:

$$1/\sqrt{2}\left(|\text{hard}\rangle_1|\psi_a(x)\rangle_1|\text{soft}\rangle_2|\psi_f(x)\rangle_2 - |\text{soft}\rangle_1|\psi_a(x)\rangle_1|\text{hard}\rangle_2|\psi_f(x)\rangle_2\right)$$

- then the statistical postulate and Bohm's equations will reproduce all the QM predictions about EPR states.
  - So by Bell's theorem nonlocality had to show up somewhere. Let's examine it...

Bohmian nonlocality

- If e1 goes through then e1 will be effectively hard and e2 will be effectively soft.



- If (instead) e2 goes through a (right-side-up) hardness box then e2 will be effectively hard and e1 effectively soft.

- If (instead) e1 goes through an *upside-down* hardness box then e1 is effectively soft, e2 effectively hard.
  - This will enable faster than light *communication*.

Bohmian mechanics and relativity

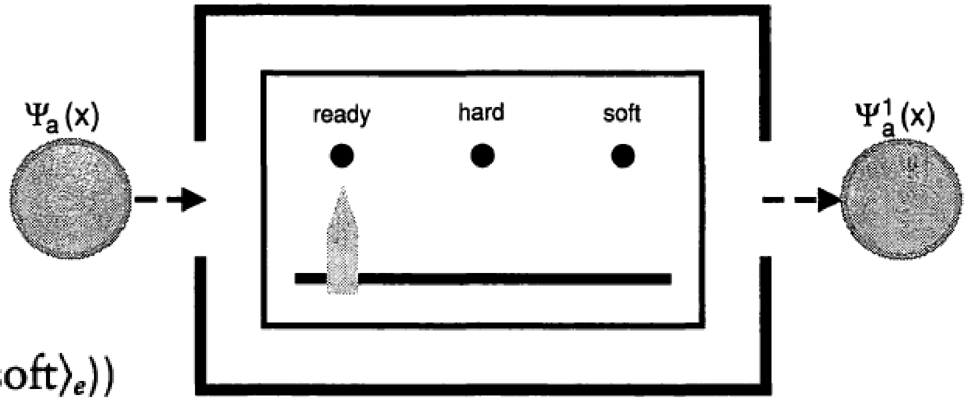
- Bohmian mechanics therefore requires an absolute standard of simultaneity that defines the simultaneous sending/receiving of information (over any distance)
- Insofar as relativity rules this out Bohmian mechanics is inconsistent with relativity theory.

- But we can only send this information if we know the positions of the particles.
  - If we only know the wave-function then there is no effective difference from standard QM.
  - This allows Bohmians to treat relativity instrumentally: it's false but (given our ignorance of particle positions) it's as if it's true.

The appearance of collapse

Appearance of collapse

- Send black e through this device:
- State evolution:



$$|\psi_r\rangle_m(|\psi_a\rangle_e|\text{black}\rangle_e) \rightarrow$$

$$1/\sqrt{2}(|\psi_h\rangle_m(|\psi_b\rangle_e|\text{hard}\rangle_e) + |\psi_s\rangle_m(|\psi_b\rangle_e|\text{soft}\rangle_e))$$

  - $|\psi_r\rangle_m$, $|\psi_h\rangle_m$, and $|\psi_s\rangle_m$ are 3 distinct states for the billions of particles composing the pointer and $|\psi_a\rangle_e$ and $|\psi_b\rangle_e$ are coordinate space states of the depicted electron.
- Once all the particles are on one of the terms there is effective collapse.
  - Unless the two branches drift back together!
  - Suppose they (somehow) do except that an air molecule becomes correlated with the pointer and hence the electron.
  - Until the coordinate space air molecule wave-function drift back together the electron is still effectively collapsed.

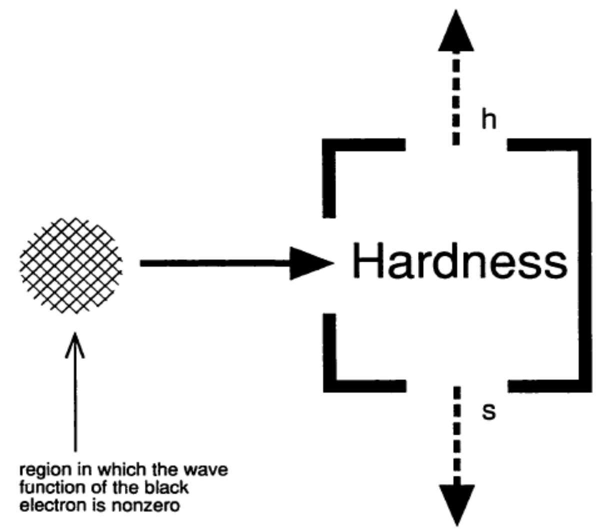
Appearance of collapse

- The problem is that the environment of a pointer acts as a giant collection of measuring devices for the pointer's position.
- Confirming that the Bohm wave-function hasn't collapsed will involve either avoiding or reversing or taking into account every last one of those measurements.

- So Bohm's theory makes it look as if wave-functions collapse during any measurement in accordance with the QM collapse postulate.
  - At least: if we only know the particle's wave-function (or the effective wave-function).
  - Let's see how that works...

Appearance of collapse

- Consider electron with w-f:
  - |black›|ψ_a(x)›
- From this w-f we can infer that a colour measurement will yield *black*.



- If we also knew it was in the upper region of ψ_a(x) then we could also infer that a hardness measurement would yield *hard* in violation of the uncertainty relations.

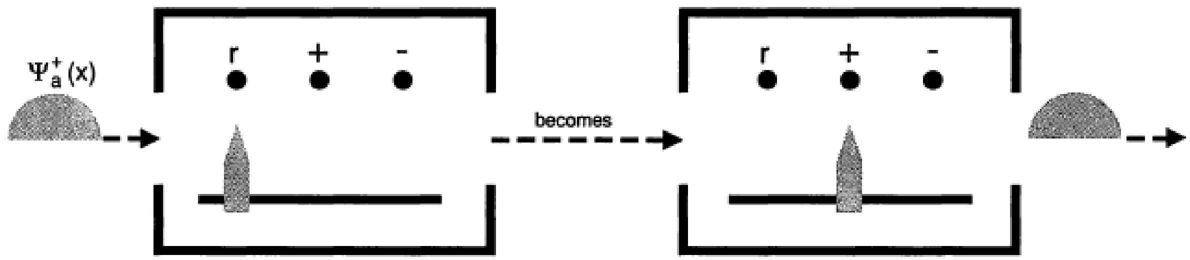
- But how *could* we know the position of the electron?

Appearance of collapse

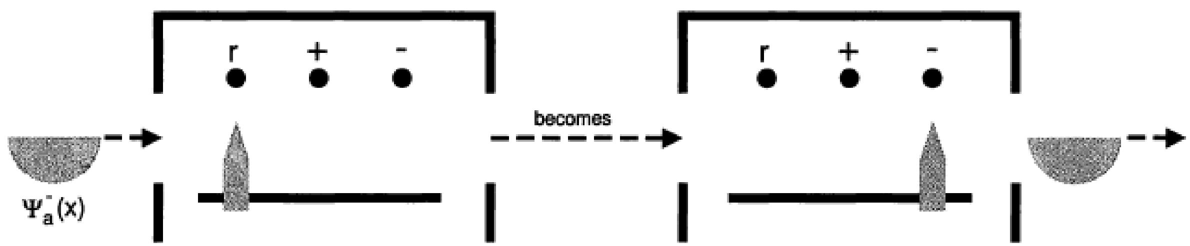
- To discover whether the electron is in the upper or lower half we would have to measure its position.
- But then we would correlate the electron's wave-function to the wave-function of the measuring device.
- So finding out whether the electron is in the upper or lower half of the region in which $\psi_a(x)$ is nonzero "region a" without changing that wave-function is impossible.
- Let's illustrate...

Appearance of collapse

- If we try to measure where in region a the electron is using:



- Then the joint electron-device w-f will evolve as follows:



$$|\mathrm{r}\rangle_m|\psi_a(x)\rangle_e|\mathrm{black}\rangle_e \rightarrow$$
$$|+\rangle_m|\psi_a^+(x)\rangle_e|\mathrm{black}\rangle_e + |-\rangle_m|\psi_a^-(x)\rangle_e|\mathrm{black}\rangle_e$$

Appearance of collapse

- So we have:

$$|+\rangle_m |\psi_a^+(x)\rangle_e |\text{black}\rangle_e + |-\rangle_m |\psi_a^-(x)\rangle_e |\text{black}\rangle_e$$

- Where + means upper half, - means lower half.
  - Imagine also 1/√2 amplitudes (Albert has left them off).
- This nonseparability now changes the way the future motions depend on its present position.
  - If e's coordinate space w-f effectively collapses to + then the outcome of an upcoming hardness measurement will depend on *where in the top half* e is located!
    - 0.5 prob for top quarter, 0.5 prob for next-to-top quarter.
    - Follows from Bohm's laws.
  - Bohm's law's conspire to insure that the position measurement will do no good at all.

Appearance of collapse

- Bohm's theory entails that if we initially know nothing of a system but its w-f then all we can know at any future time of that system's location will be what follows from its effective w-f at that future time.

- Applied to the universe: all that can be found out about the present state of any system can be summed up in a w-f.
  - Sometimes an actual one, mostly an effective one.

- So we cannot predict future states with any more accuracy than is allowed by wave-functions alone.

Appearance of collapse

- “That’s how this theory manages to clean up after itself [...] And so if this theory is right then the fundamental laws of the world are cooked up in such a way as to systematically mislead us about themselves” (Albert p169).

Objections

Objections

- Albert on mentality
  - It is unclear whether the theory guarantees that every measurement has a outcome.
- Albert on Incommensurability with Many-Minds dualism.
  - No possible experimental means of deciding between them.
- Incompatibility with special relativity
  - There is a preferred reference frame although no experiment can determine which frame it is.
- On whether the “conspiracy theory” is too ad hoc
  - Bohm has designed the laws to make it look as if QM is true.
- The status of the wave-function
  - It’s a field that pushes particles around?
  - But decoherent branches seem structurally similar to the actual particle configuration – Many worlds in denial?
    - See readings for next week on eLearning.

- We shall discuss these next week and wrap up the course.