

Philosophy of quantum mechanics

VU University Amsterdam: W_MASP_TF013 Lecture 3: 10/2/2015

Kelvin J. McQueen

Today's Lecture

- ▶ Recap from previous lectures.
- ▶ Discussion of the second half of “The Mathematical Formalism”.
 - ▶ Complex vector spaces.
 - ▶ Hermitian operators.
 - ▶ Other spin space properties.
 - ▶ Degrees of incompatibility.
 - ▶ Coordinate space.
 - ▶ Composite (e.g. 2-particle) systems.
 - ▶ Applying the formalism to the experiments.
- ▶ The dimensionality of space



Recap from previous lecture



Recap

- ▶ So far we have seen that:
- ▶ Quantum mechanics appeals to “measurement” to determine when its indeterministic law applies.
 - ▶ This is philosophically problematic. So we need to understand where appeal to “measurement” arises in the formalism.
- ▶ Experiments on particles (e.g. electrons) cannot (easily) be understood in terms of any (logically possible) classical descriptions; and so we require new concepts.
 - ▶ Objective probability, superposition, and incompatibility.
 - ▶ So we need to understand how quantum mechanics formalises these concepts.

Recap of the formalism

- ▶ Quantum mechanics (under the standard interpretation) can be broken down into 5 principles:
 - ▶ Physical states
 - ▶ Represented by certain kinds of vectors.
 - ▶ Measureable properties
 - ▶ Represented by certain kinds of operators.
 - ▶ Dynamics
 - ▶ Schrödinger equation changes vector direction over time, deterministically and linearly.
 - ▶ The connection with experiment
 - ▶ The eigenstate-eigenvalue link...
 - ▶ Collapse
 - ▶ Measurement of the property represented by operator \hat{O} when the measured system is not in an eigenstate of \hat{O} will collapse the system into such an eigenstate, with a certain probability.

The connection with experiment

- ▶ Eigenstate-eigenvalue link
 - ▶ A state possesses the value v of a property represented by operator O if and only if that state is an eigenstate of O with eigenvalue v .
- ▶ Properties (e.g. colour) are represented by operators.
- ▶ Physical states are represented by vectors.
- ▶ A state possesses a value of a property (e.g. white) if the vector representing that state is an eigenvector of that property's operator.
- ▶ If $O|white\rangle$ yields $\#|white\rangle$ then $|white\rangle$ is an eigenvector of O with eigenvalue $\#$.
- ▶ The eigenvalue tells us the value of that property i.e. The physical state of that system.

Collapse and objective probability

- ▶ If a physical system is not *in* an eigenstate of operator O ...
 - ▶ That is, if the system's state is not *represented* by an eigenvector of O ...
- ▶ And if we measure the system in the hope of finding a value for the property represented by O ...
- ▶ Then the state collapses, with a certain objective probability, to some value for that property.
 - ▶ Example: the probability that a hardness measurement on a *white* electron yields hard:

$$\begin{aligned} &= (\langle \text{white} | \text{hard} \rangle)^2 \\ &= \left(\frac{1}{\sqrt{2}}\right)^2 \end{aligned}$$

Interpretations of quantum mechanics

- ▶ This is the Copenhagen interpretation of quantum mechanics.
 - ▶ A.K.A. The “standard view”.
 - ▶ Due to overwhelming acceptance among physicists throughout 20th century. Still the most widely accepted according to a 1997 poll. More recent polls have given varying results.
- ▶ Key features:
 - ▶ Measurement brings the world into meaningful states (i.e. states we can meaningfully talk about).
 - ▶ “Measurement” is not explained – taken as primitive.
 - ▶ Nonetheless: most successful algorithm for predicting natural phenomena ever invented.

Realism about quantum mechanics

- ▶ Quantum mechanics describes a mind-independent reality.
- ▶ But what is the reality described by quantum mechanics?
- ▶ What is the ontology of quantum mechanics?

Discussion of “The Mathematical Formalism” (2nd half)

Albert (1992: ch.2)

The mathematical formalism

- ▶ **Complex vector spaces**
 - ▶ Although we won't get into the calculations, we need to be aware of these.
- ▶ **Hermitian operators.**
 - ▶ The operators quantum mechanics actually uses, which have interesting implications e.g. for spin space properties.
- ▶ **Other spin space properties.**
 - ▶ Any vector in the space represents a value for some property – I'll describe some.
- ▶ **Degrees of incompatibility.**
 - ▶ We can quantify the extent to which properties are incompatible.
- ▶ **Coordinate space**
 - ▶ Representing the continuous property of position.
- ▶ **Composite (e.g. 2-particle) systems.**
 - ▶ Essential for understanding entanglement (today) and nonlocality (Thursday).
- ▶ **Applying the formalism to the experiments...**



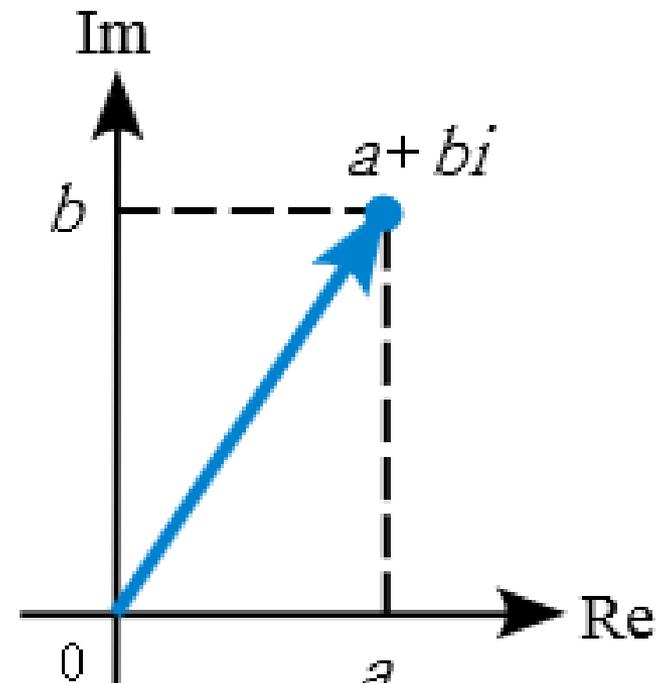
Complex vector spaces

Complex vector spaces

- ▶ In the *real vectors spaces* we have considered, we multiply vectors by real numbers to get new vectors.
- ▶ In *complex vector spaces*, we multiply vectors by complex numbers to get new vectors.
- ▶ Colour and hardness operators are defined on a 2-dimensional real vector space.
- ▶ This is an idealisation because spin operators are defined on a 2-dimensional complex vector space.

Complex numbers

- ▶ In complex vector spaces, we multiply vectors by complex numbers to get new vectors.
- ▶ Complex numbers are either real numbers or *imaginary* numbers.
- ▶ They take the form **$a+bi$**
 - ▶ a and b are real numbers, i is the imaginary number:
$$\sqrt{-1}$$
- ▶ Can be represented geometrically using the imaginary dimension:



Why does QM require complex spaces?

- ▶ Main reason: we cannot predict the outcomes of most experiments without them.
 - ▶ QM represents physical states (like being white) with length-1 vectors in a given vector space.
 - ▶ The dynamics describes changes in physical systems by rotating the state vector in the given vector space.
 - ▶ The dynamics rotates state vectors through the imaginary plane.
 - ▶ The justification for this is that it works (i.e. yields probability amplitudes that correctly predict statistical outcomes).

Determining probabilities

- ▶ Let $|B=bi\rangle$ denote the eigenvector of property B with eigenvalue b_i .

- ▶ Thus $|\text{Colour}=-|\rangle$ denotes $|\text{white}\rangle$.

- ▶ In \mathbb{R}^2 if $|A\rangle$ is the state vector of a system then the probability that a B measurement gives $|B=bi\rangle$ is:

$$(\langle a|B = bi \rangle)^2$$

- ▶ But in \mathbb{C}^2 this formula may yield imaginary numbers rather than probabilities.

- ▶ So we must determine *the absolute value* before squaring:

$$|\langle a|B = bi \rangle|^2$$

Determining probabilities

- ▶ Consider the complex number:

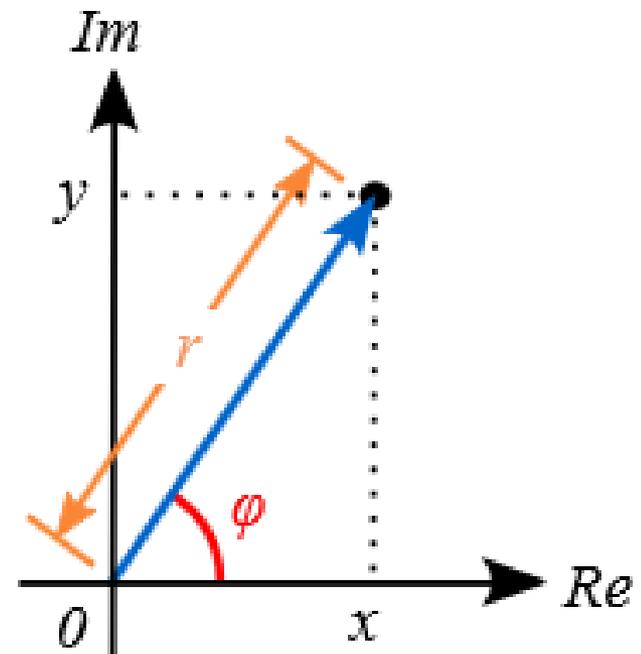
$$z = x+yi$$

- ▶ The absolute value (or modulus) of z is:

$$r = |z| = \sqrt{x^2 + y^2}$$

- ▶ We now square $|z|$ to obtain a probability.

- ▶ This operation is called **The Born Rule** after Max Born.



Will we use complex vector spaces?

- ▶ Complex vector spaces are not essential for understanding the measurement problem.
- ▶ Assessment will therefore not concern calculations with imaginary numbers.
- ▶ It is just important to be aware of them!



Hermitian operators



Hermitian operators

- ▶ An operator on a complex space is (still) an instruction to transform vectors into other vectors, but matrix elements are complex, e.g.

$$\begin{bmatrix} 2 & 1 - i \\ 1 + i & 3 \end{bmatrix}$$

- ▶ Our matrix equations remain the same.
- ▶ But we need operators that always give real eigenvalues: Hermitian operators.

Hermitian operators – key properties

- ▶ 1. Eigenvectors (of some operator) with distinct eigenvalues are always orthogonal.
- ▶ 2. Eigenvectors (of some operator on some space) always form a basis (for that space).
- ▶ 3. Distinct eigenvalues have unique eigenvectors
- ▶ 4. Unique physical property for each Hermitian operator.
- ▶ 5. Any vector is an eigenvector of some Hermitian operator.
 - ▶ Let's draw out implications of 4 & 5...

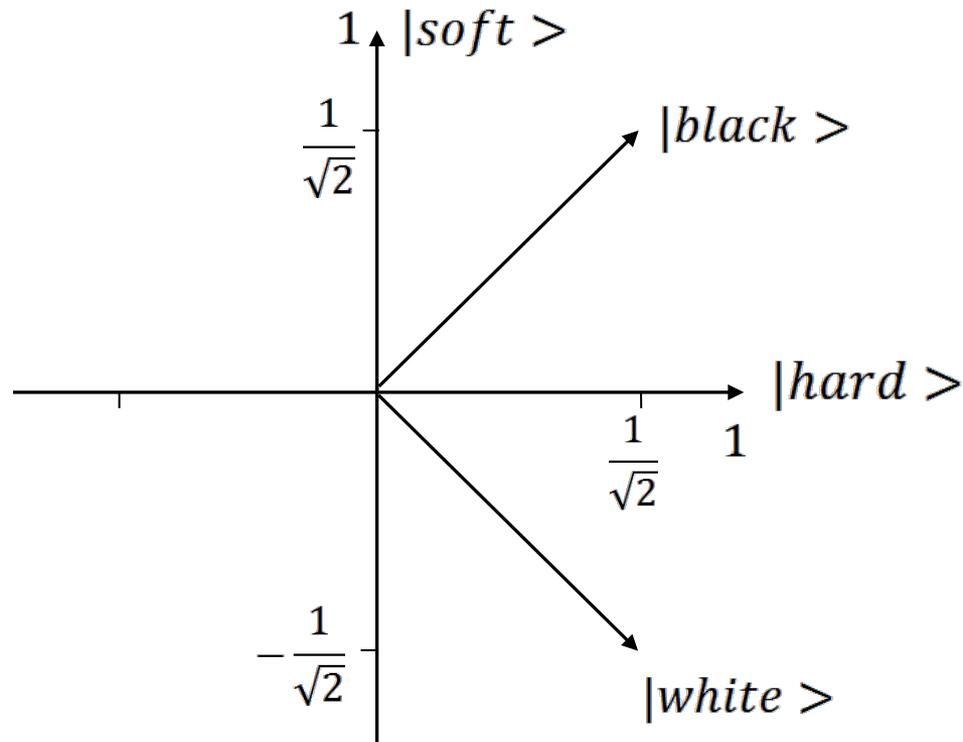
Infinitely many measurable properties

- ▶ Any Hermitian operator on a given space is associated with some measurable property.
- ▶ Any vector in a given space will be an eigenvector of some Hermitian operator.
- ▶ But vector spaces (of more than one dimension) have an infinity of (length-1) vectors.
- ▶ So physical systems have an infinity of mutually incompatible measurable properties.
- ▶ This can be illustrated on the two dimensional real space used to represent hardness and colour vectors...

Other spin space properties

Other spin space properties

► Recall:



- What about all the other vectors in this space? What states do they represent? What properties are such states values of?

Other spin space properties

▶ Let's just make up some names:

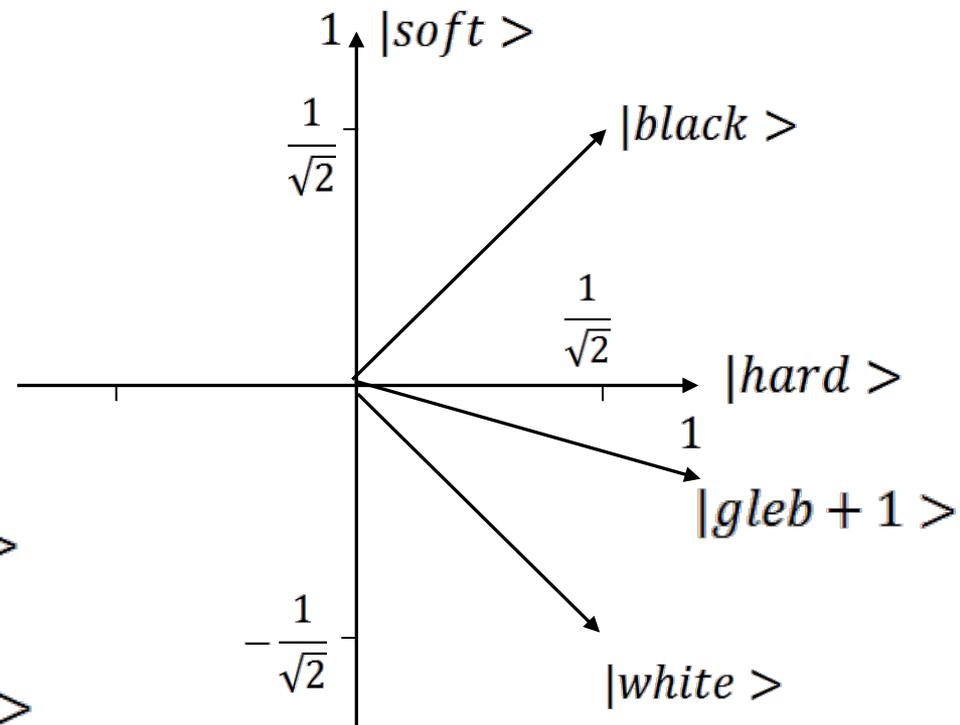
- ▶ Let “gleb” be the property that has orthonormal eigenvectors $|\text{gleb}+1\rangle$ and $|\text{gleb}-1\rangle$.

▶ Then we have

(in the colour basis):

$$|\text{gleb} + 1\rangle = \frac{1}{2} |\text{black}\rangle + \frac{\sqrt{3}}{2} |\text{white}\rangle$$

$$|\text{gleb} - 1\rangle = \frac{\sqrt{3}}{2} |\text{black}\rangle - \frac{1}{2} |\text{white}\rangle$$



Other spin space properties

- ▶ We can now find a property that is incompatible with *gleb*. Again, just invent a name – “*scrad*”:

$$|scrad + 1 \rangle = \frac{1}{2}|black \rangle - \frac{\sqrt{3}}{2}|white \rangle$$

$$|scrad - 1 \rangle = \frac{\sqrt{3}}{2}|black \rangle + \frac{1}{2}|white \rangle$$

- ▶ Is *scrad* incompatible with colour?
 - ▶ Is hardness *more incompatible* with colour than *scrad*?
 - ▶ Can there be degrees of incompatibility?



Degrees of incompatibility



Incompatibility

- ▶ **Colour and hardness are incompatible.**
 - ▶ Definite colour states are superpositions of hardness (and vice versa).
- ▶ Experimentally, this means that hardness measurements on white (or black) electrons yield 50/50 results (50% hard, 50% white).

- ▶ **Colour and gleb are also incompatible.**
 - ▶ Definite colour states are superpositions of gleb (and vice versa).
- ▶ Experimentally, gleb measurements on white electrons also yield random results; but not 50/50 results...

Degrees of incompatibility

- ▶ What is the probability that a colour measurement on a $g_{leb}+1$ electron will yield white?

$$\begin{aligned} & (\langle a|B = bi \rangle)^2 \\ &= (\langle g_{leb} + 1|white \rangle)^2 \\ &= \left(\begin{bmatrix} \frac{\sqrt{3}}{2} \\ 1 \\ \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)^2 = \left(\frac{\sqrt{3}}{2} \times 1 + \frac{1}{2} \times 0 \right)^2 = \left(\frac{\sqrt{3}}{2} \right)^2 \\ &= 0.75 \end{aligned}$$

Degrees of incompatibility

- ▶ So, the probability of finding a $g_{\text{leb}}+I$ electron to be white is 0.75.
 - ▶ Hence the probability of finding a $g_{\text{leb}}+I$ electron to be black is 0.25.
- ▶ Compare: the probability of finding a hard electron to be white is 0.5.
 - ▶ Hence the probability of finding a soft electron to be white is 0.5.
- ▶ Hence, g_{leb} and colour are *more compatible* than hardness and colour.
 - ▶ G_{leb} eigenstates do not randomise colour measurement results to the same degree that hardness eigenstates do.

Degrees of incompatibility

- ▶ A slightly more complex (but more general) way of determining the degree of incompatibility between two properties A and B uses the commutator of A and B:
 - ▶ $[A,B] = AB - BA$
 - ▶ On the left hand side we are subtracting one matrix product from another (we won't worry about the rules for doing this).
 - ▶ If $[A,B] \neq 0$ then operators A and B have no eigenvectors in common (as with colour, hardness, gleb, etc.).
- ▶ Momentum P and position X are incompatible such that:
 - ▶ $[P,X] = PQ - QP = \frac{h}{2\pi i} I$
 - ▶ Where I is the identity operator and $\frac{h}{2\pi}$ is planck's constant or "h-bar".
 - ▶ This symbol is missing from Albert's equation 2.24 on page 44!



Coordinate space



Position and momentum

- ▶ Position is maximally incompatible with momentum.
 - ▶ Definite position states are superposed momentum states (and vice versa).
 - ▶ Momentum (denoted p) equals mass times velocity.
- ▶ Our formalism must represent incompatibility relations.
 - ▶ So we need:
 - ▶ A position operator and a momentum operator.
 - ▶ A corresponding state space with a basis consisting entirely of position eigenvectors and a basis consisting entirely of momentum eigenvectors.

Representing continuous properties

- ▶ Because colour and hardness are binary properties, we could represent their states in 2-dimensional state space.
- ▶ But position and momentum are continuous properties: values for such properties fall on an infinite continuum.
- ▶ Consider a *one* dimensional *physical* space; points in that space form a line.
 - ▶ Each point in that line is a possible position state.
 - ▶ So each point on that line is an eigenstate of position represented by a unique eigenvector of the position operator.
 - ▶ These eigenvectors form a basis of the space.
- ▶ But that's an infinite number of basis vectors!

Representing continuous properties

- ▶ **An example:**

- ▶ Let X denote the position operator.
- ▶ Let $|X=2\rangle$ denote the eigenvector representing location “2”.
- ▶ Let $|\psi\rangle$ be some vector in the coordinate space...

- ▶ $|\psi\rangle = \#1|X=1\rangle + \#2|X=2\rangle \dots \#n|X=n\rangle$

- ▶ If the space is genuinely continuous this sum should be an integral.

- ▶ **But typically we can idealise, and pretend that space is discrete and finite.**

- ▶ E.g. see Albert’s 4D space operator, eqn 2.15 p29.

Wave functions

- ▶ ψ is a symbol you will see often...
 - ▶ Let $\psi(x)$ denote a function from position (X) values to $\#$ values.
 - ▶ So $\psi(x)$ is like an infinite list of correspondences between X values and $\#$ values.
- ▶ $\psi(x)$ functions are called *wave functions*.
- ▶ Any measurable properties of particles (momentum, energy etc.) can be represented as an operator on the wave-function.

Wave functions

- ▶ Input position states and it outputs # distributions.
 - ▶ The wave function for $|X=5\rangle$ assigns 1 to position 5 and distributes 0 elsewhere.
 - ▶ The wave function for $\#_1|X=1\rangle + \#_2|X=2\rangle$ assigns $\#_1$ to position 1 and $\#_2$ to position 2, 0 elsewhere.
- ▶ Determining probabilities
 - ▶ If the wave function at a time is $\psi(x)$ then the probability of finding the particle at location x is $|\psi(x)|$ squared.
- ▶ In general, everything said about state vectors can be translated into the language of wave functions.



Composite systems

Composite systems

- ▶ So far we've only discussed measurements on single particles.
- ▶ As we shall see, measurements on multiple particles in certain conditions produces very strange results.
 - ▶ Non-separable, or entangled particles.
- ▶ If particle 1 is in state $|\psi_a\rangle$ and particle 2 is in state $|\psi_b\rangle$ then the state of their composite is written:
- ▶ $|\psi_a\rangle_1 |\psi_b\rangle_2$

Non-interacting particles

- ▶ If the two particles don't interact the joint probability that the outcome of a measurement of A on particle 1 is a and that the outcome of measurement B on particle 2 is b, is:
 - ▶ The probability of the former outcome (taken alone) times the probability of the latter outcome (taken alone).
- ▶ Dimensionality of the two-particle state space is the product of the dimensionalities of the state spaces of the individual particles.
 - ▶ A single vector in such a space will represent the joint state of the two particle system.
 - ▶ Basis vectors represent two-particle states.

Non-separable states

- ▶ The eigenvectors of operators that represent properties of two-particle composites form orthonormal bases of the state space.
- ▶ This means that two-particle states will sometimes be represented by weighted sums of such vectors. Albert's example:

$$|Q'\rangle = \frac{1}{\sqrt{2}} |X^1 = 5\rangle |X^2 = 7\rangle + \frac{1}{\sqrt{2}} |X^1 = 9\rangle |X^2 = 11\rangle$$

- ▶ Superscripts name particles (e.g. particle "1").
- ▶ So particle 1 (superposed between positions 5&9) is entangled with particle 2 (superposed between positions 7&11).

Non-separable (entangled) states

$$|Q'\rangle = \frac{1}{\sqrt{2}} |X^1 = 5\rangle |X^2 = 7\rangle + \frac{1}{\sqrt{2}} |X^1 = 9\rangle |X^2 = 11\rangle$$

- ▶ The composite is in a superposition of being at (5,7) and being at (9,11).
- ▶ The strange aspect of such states is that if we were to measure the position of particle 1, we would not only collapse it to either 5 or 9 (with 50/50 probabilities); we would *also* collapse particle 2 along with it (instantaneously, no matter how far away it is).
- ▶ We will examine this property in detail in the next lecture.

Non-separability of distinct properties

- ▶ Just as the positions of *two* particles can be non-separable, so too can the position and (say) hardness of a *single* particle.
- ▶ For example:

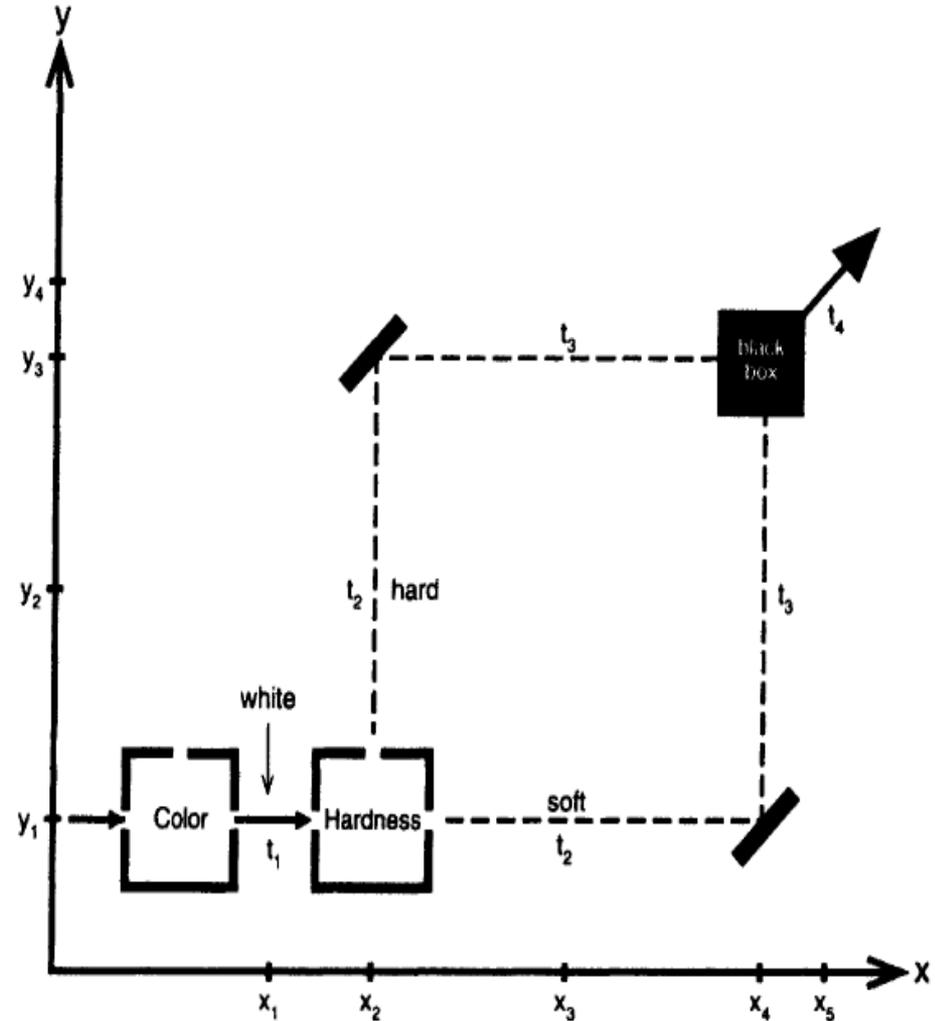
$$|Q'' \rangle = \frac{1}{\sqrt{2}} |hard \rangle |X = 5 \rangle + \frac{1}{\sqrt{2}} |soft \rangle |X = 9 \rangle$$

- ▶ This is crucial to understanding the difference between 2-path experiments 3 and 4.

The two-path experiments

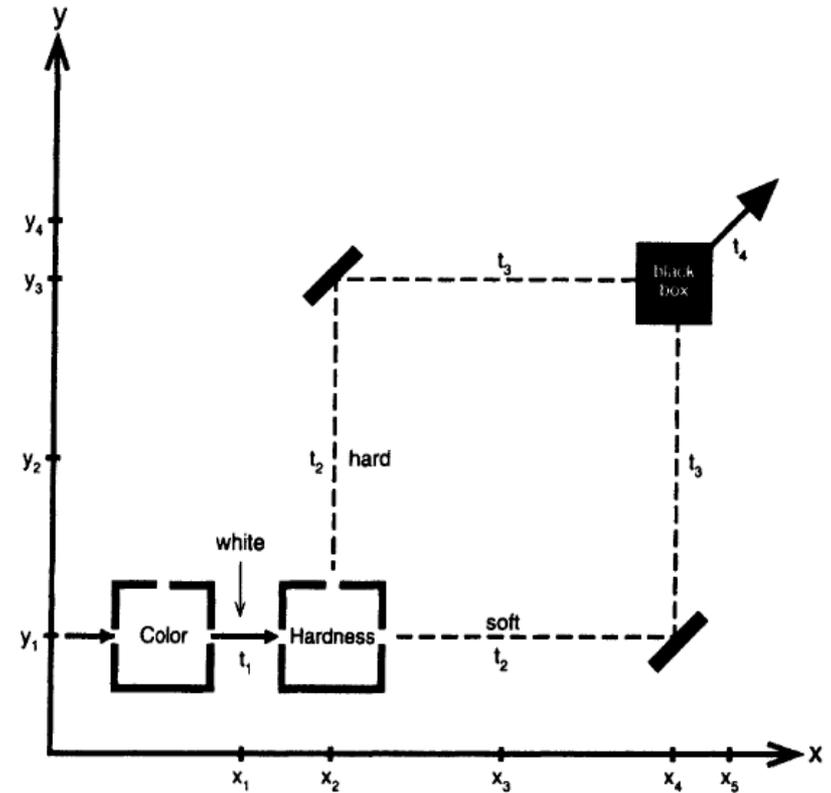
The 2-path experiments

- ▶ This is experiment 3 where we send in white electrons and measure colour.
- ▶ We want to know why we get all white and why adding a wall to a path randomises the results.
- ▶ The position co-ordinates are crucial here.



The 2-path experiments

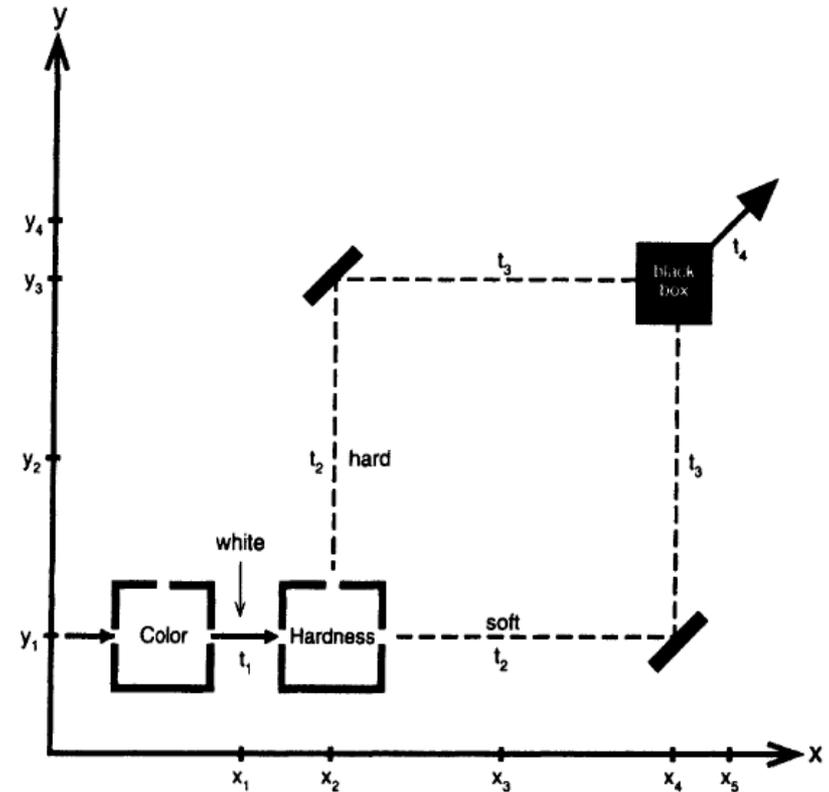
- ▶ At time t_1 the electron is white and is located at (x_1, y_1) .
- ▶ We can represent that state in the hardness basis as follows (note the *separability* of hardness and position):



$$|t_1 \text{ state} \rangle = \frac{1}{\sqrt{2}} |hard \rangle |X = x_1, Y = y_1 \rangle - \frac{1}{\sqrt{2}} |soft \rangle |X = x_1, Y = y_1 \rangle$$

The 2-path experiments

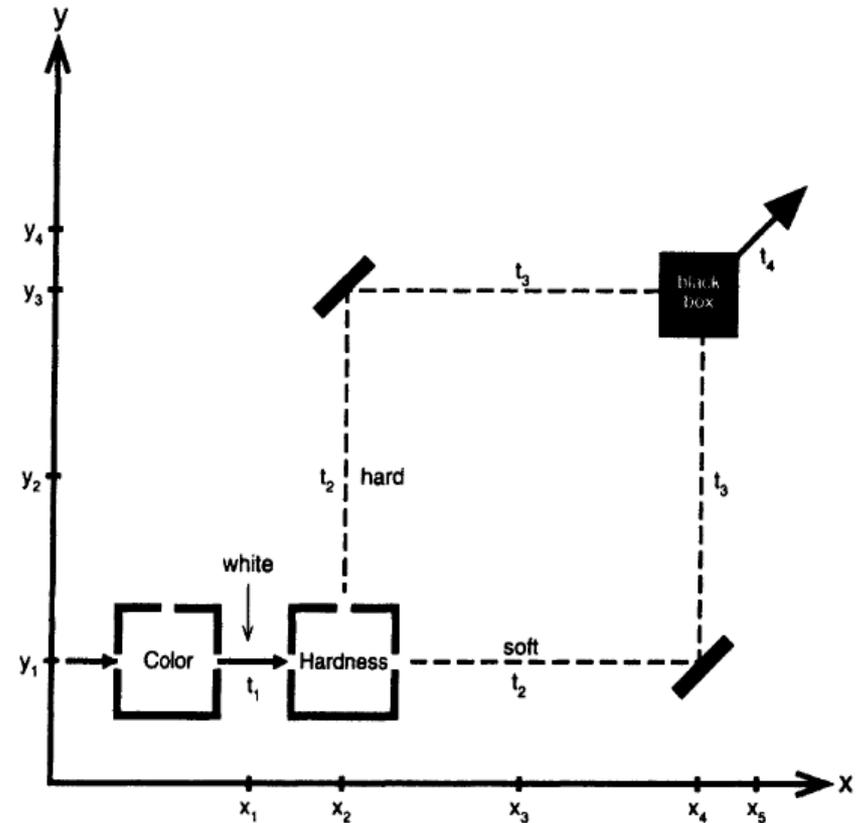
- ▶ At time t_2 the electron is in a superposition of going down the h and s paths.
- ▶ We can represent its state as follows (note the *non-separability* of hardness and position):



$$|t_2 \text{ state} \rangle = \frac{1}{\sqrt{2}} |hard \rangle |X = x_2, Y = y_2 \rangle - \frac{1}{\sqrt{2}} |soft \rangle |X = x_3, Y = y_1 \rangle$$

The 2-path experiments

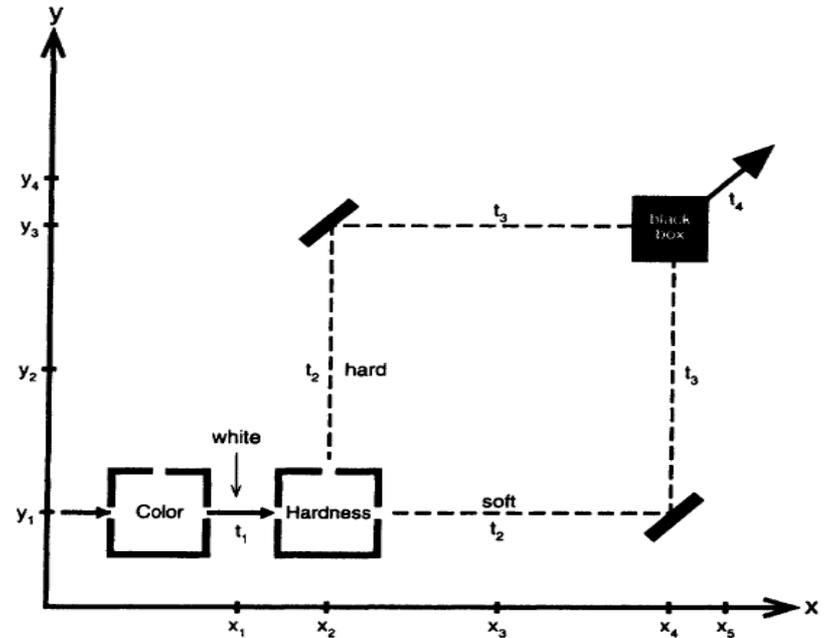
- ▶ At t_3 the electron is reflected off the mirrors but remains in the non-separable state...



$$|t_3 \text{ state} \rangle = \frac{1}{\sqrt{2}} |hard \rangle |X = x_3, Y = y_3 \rangle - \frac{1}{\sqrt{2}} |soft \rangle |X = x_4, Y = y_2 \rangle$$

The 2-path experiments

- ▶ At t_4 the electron beams are recombined so that hardness and position become separable.
- ▶ *This is why we get all white!*

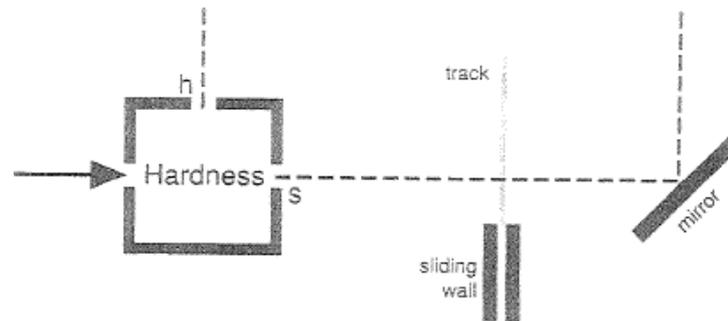


$$\begin{aligned}
 |t_4 \text{ state} \rangle &= \frac{1}{\sqrt{2}} |hard \rangle |X = x_5, Y = y_4 \rangle - \frac{1}{\sqrt{2}} |soft \rangle |X = x_5, Y = y_4 \rangle \\
 &= \left(\frac{1}{\sqrt{2}} |hard \rangle - \frac{1}{\sqrt{2}} |soft \rangle \right) |X = x_5, Y = y_4 \rangle \\
 &= |white \rangle |X = x_5, Y = y_4 \rangle
 \end{aligned}$$

The 2-path experiments

- ▶ Now let's add a sliding wall. The t_2 state will be the same non-separable state:

$$|t_2 \text{ state} \rangle = \frac{1}{\sqrt{2}} |hard \rangle |X = x_2, Y = y_2 \rangle - \frac{1}{\sqrt{2}} |soft \rangle |X = x_3, Y = y_1 \rangle$$

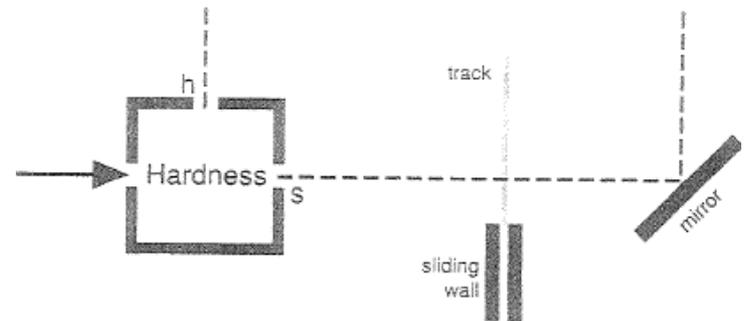


- ▶ But the t_3 state will be:

$$|t_3 \text{ state} \rangle = \frac{1}{\sqrt{2}} |hard \rangle |X = x_3, Y = y_3 \rangle - \frac{1}{\sqrt{2}} |soft \rangle |wall! \rangle$$

The 2-path experiments

- ▶ Since the wall causes the $t3$ state to be:



$$|t3\ state\ \rangle = \frac{1}{\sqrt{2}} |hard\ \rangle |X = x3, Y = y3\ \rangle - \frac{1}{\sqrt{2}} |soft\ \rangle |wall!\ \rangle$$

- ▶ The $t4$ state remains non-separable and so is not decomposable into a definite colour state:

$$|t4\ state\ \rangle = \frac{1}{\sqrt{2}} |hard\ \rangle |X = x5, Y = y4\ \rangle - \frac{1}{\sqrt{2}} |soft\ \rangle |wall!\ \rangle$$

- ▶ Colour measurements will therefore collapse this state explaining the 50/50 result.

The 2-path experiments

- ▶ Although the $t4$ state...

$$|t4\ state\rangle = \frac{1}{\sqrt{2}} |hard\rangle |X = x5, Y = y4\rangle - \frac{1}{\sqrt{2}} |soft\rangle |wall!\rangle$$

- ▶ ...is not an eigenstate of colour (or hardness), it *is* an eigenstate of some other operator.
 - ▶ Albert's "Zap" operator (eqn 2.60, p58).



Dimensionality

Realism about dimensionality

- ▶ Kant (Critique of Pure Reason) was an anti-realist about space and its properties.
 - ▶ Space and its three-dimensionality is *necessarily imposed* onto our experiences by cognition.
 - ▶ Reality itself (noumena) is not spatial and so cannot be said to have spatial dimensions.
 - ▶ Supposedly explains why Euclidean geometry seems synthetic (true in virtue of the world) but necessary and a priori.
- ▶ But modern physics denies that physical space is Euclidean (e.g. axiom of parallels is denied).
- ▶ *Realism about dimensionality* means taking physics as the authority as to the nature of space and its dimensions.

Dimensionality

- ▶ Realism about space means being prepared to accept one of many different hypotheses about how many dimensions space has.
 - ▶ Space *looks* three-dimensional (3D).
 - ▶ Space is 3D according to classical Newtonian physics.
 - ▶ Space is an aspect of a 4D spacetime in Einstein's theories of relativity.
 - ▶ This replaced a 1914 attempt to unify electromagnetism with general relativity by postulating 4D space (plus 1D time).
 - ▶ Space is 10D in one version of string theory (11D in another).
- ▶ But realism still poses constraints.
 - ▶ Consistency with *the appearance* of 3D space.

Consistency with 3D space

- ▶ Smolin (2006) describes one way of achieving consistency:
 - ▶ “we can make the new dimension a circle, so that when we look out, we in effect travel around it and come back to the same place. Then we can make the diameter of the circle very small, so that it is hard to see that the extra dimension is there at all. To understand how shrinking something can make it impossible to see, recall that light is made of waves and each light wave has a wavelength ... The wavelength of a light wave limits how small a thing you can see, for you cannot resolve an object smaller than a wavelength of the light you use to see it.”
- ▶ The dimensions can be too “small” to see!

Dimensionality in QM

- ▶ According to Alyssa Ney (2012: sec. I):
 - ▶ (A) “On any straightforward ontological reading of quantum mechanics, the theory requires the existence of an object, the wavefunction [...]
 - ▶ (B) The wavefunction inhabits an extremely high-dimensional space: configuration space. [...]
 - ▶ (C) No three of the many dimensions of configuration space correspond in any direct way with the three dimensions of our manifest image.”

Ney's argument for (A) and (B)

- ▶ (1) The laws of quantum mechanics permit the evolution of systems into entangled states.
- ▶ (2) Entangled states cannot be adequately characterised as states of something in 3D space, they are states of something spread out in a high-D space.
- ▶ (3) Therefore, there exists something that must be characterised as spread out in a high-D space.
 - ▶ Call this thing the “wavefunction”.
 - ▶ The resulting view is known as “wavefunction realism”.

Premise (1): entangled states

- ▶ Quantum state of particle located at $(4,0,0)$ where (x,y,z) are three spatial dimensions:

$$\Psi_1 = |(4, 0, 0)\rangle$$

- ▶ Quantum state of a particle in a superposition of being at $(4,0,0)$ and $(7,0,0)$:

$$\Psi_2 = \sqrt{\frac{1}{2}} |(4, 0, 0)\rangle + \sqrt{\frac{1}{2}} |(7, 0, 0)\rangle$$

- ▶ By the Born rule: 0.5 probability that measuring the particle's position will yield $(4,0,0)$, 0.5 probability for $(7,0,0)$.

Premise (1): entangled states

- ▶ Quantum state of two-particle composite in superposition of being located at (4,0,0)/(7,0,0) and at (7,0,0)/(4,0,0).

$$\Psi_3 = \sqrt{\frac{1}{2}} |(4, 0, 0)\rangle_1 |(7, 0, 0)\rangle_2 + \sqrt{\frac{1}{2}} |(7, 0, 0)\rangle_1 |(4, 0, 0)\rangle_2$$

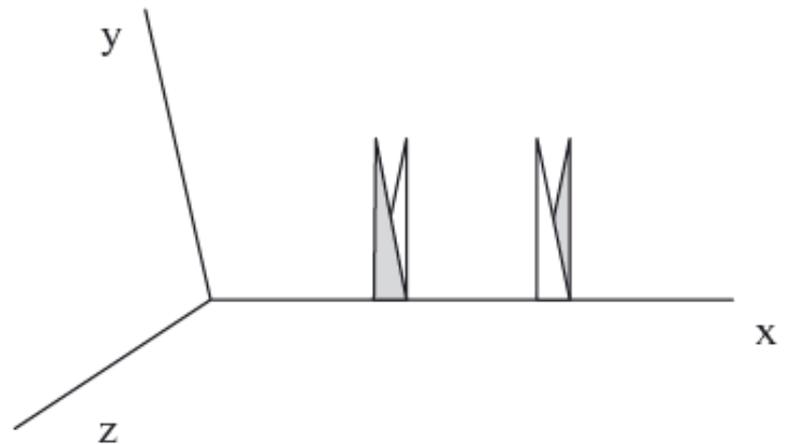
- ▶ **By the Born rule:**
 - ▶ 0.5 probability of finding the composite at (4,0,0)/(7,0,0) and 0.5 probability of finding it at (7,0,0)/(4,0,0).
 - ▶ Probability 1 for finding the two particles to be 3 units of length apart on the x-dimension.
- ▶ This is an entangled state.

Premise (2): dimensionality

- ▶ How should one describe this state:

$$\Psi_3 = \sqrt{\frac{1}{2}} |(4, 0, 0)\rangle_1 |(7, 0, 0)\rangle_2 + \sqrt{\frac{1}{2}} |(7, 0, 0)\rangle_1 |(4, 0, 0)\rangle_2$$

- ▶ ...as being instantiated in a 3D space?
 - ▶ Ney suggests representing at each point in the space the probabilities that a particle is at that location by peaks:
 - ▶ Peaks for particle 1 are grey.
 - ▶ Peaks for particle 2 are white.



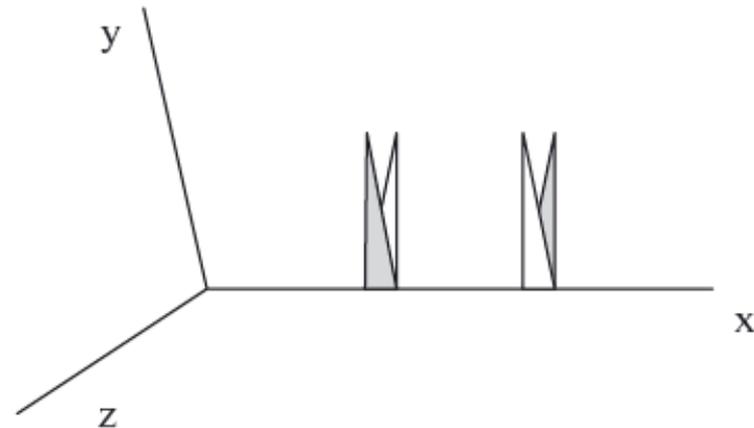
Premise (2): dimensionality

- ▶ Ney argues that this is an inadequate description of Ψ_3 because it is indistinguishable from other distinct states.
Compare:

$$\Psi_3 = \sqrt{\frac{1}{2}} |(4, 0, 0)\rangle_1 |(7, 0, 0)\rangle_2 + \sqrt{\frac{1}{2}} |(7, 0, 0)\rangle_1 |(4, 0, 0)\rangle_2$$

$$\Psi_4 = \sqrt{1/2} |(4, 0, 0)\rangle_1 |(4, 0, 0)\rangle_2 + \sqrt{1/2} |(7, 0, 0)\rangle_1 |(7, 0, 0)\rangle_2$$

- ▶ Both yield the same 3D graph:
- ▶ But they are physically distinct states!



Premise (2): dimensionality

- ▶ Compare (again):

$$\Psi_3 = \sqrt{\frac{1}{2}} |(4, 0, 0)\rangle_1 |(7, 0, 0)\rangle_2 + \sqrt{\frac{1}{2}} |(7, 0, 0)\rangle_1 |(4, 0, 0)\rangle_2$$

$$\Psi_4 = \sqrt{1/2} |(4, 0, 0)\rangle_1 |(4, 0, 0)\rangle_2 + \sqrt{1/2} |(7, 0, 0)\rangle_1 |(7, 0, 0)\rangle_2$$

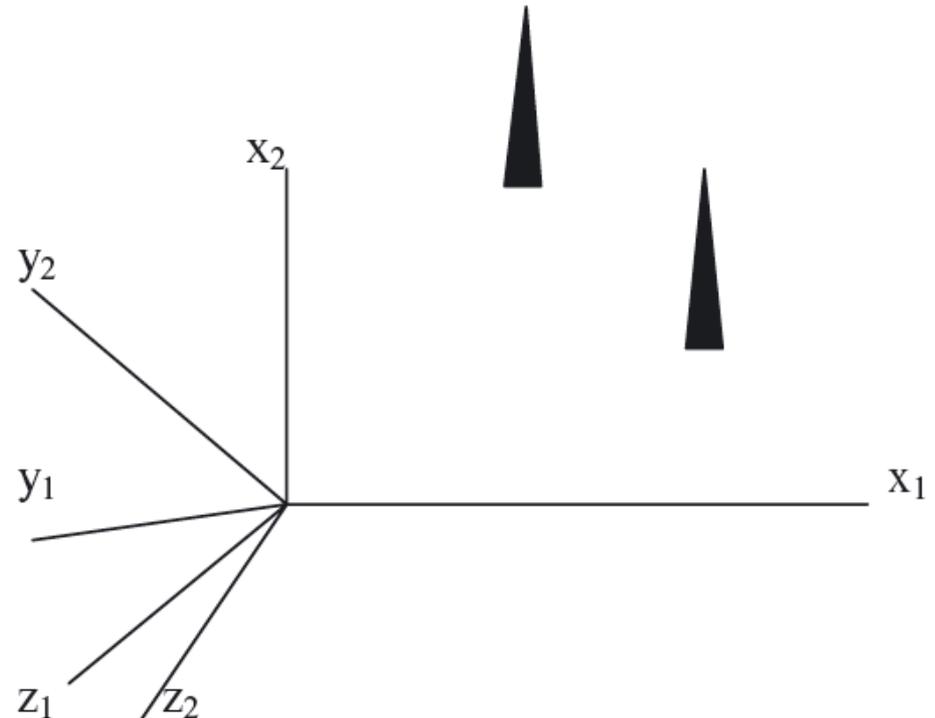
- ▶ If a system is in state Ψ_3 then the particles are **3 units apart in the x-dimension**
 - ▶ Even though neither particle is at either location.
- ▶ If a system is in state Ψ_3 then the particles are *not* **3 units apart in the x-dimension**.
 - ▶ Both particles will be found (upon measurement) to be in the same position with probability one.

Premise (2): dimensionality

- ▶ Then how can we understand states Ψ_3 and Ψ_4 as being instantiated in a physical space with more dimensions?
- ▶ According to Ney these states can be represented in a 3 times N dimensional space
 - ▶ N is the number of particles in the space.
- ▶ The 3N space is called *configuration space*.

Configuration space

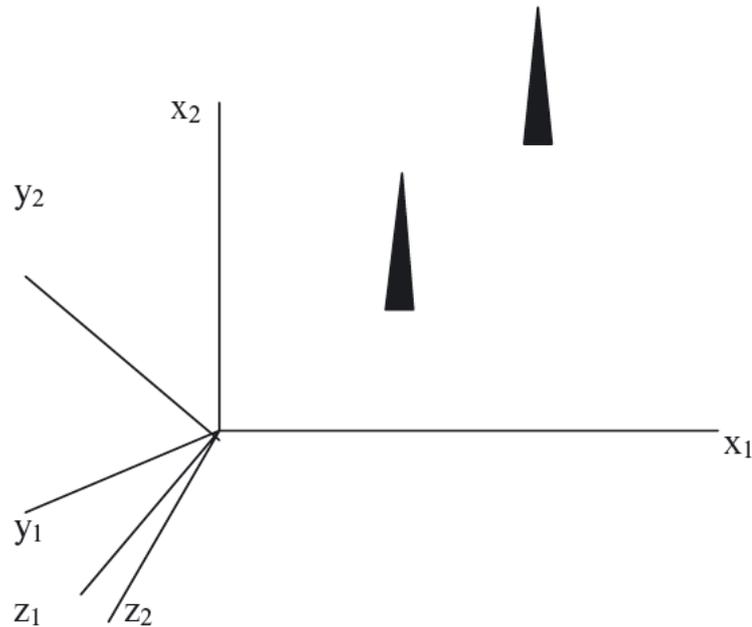
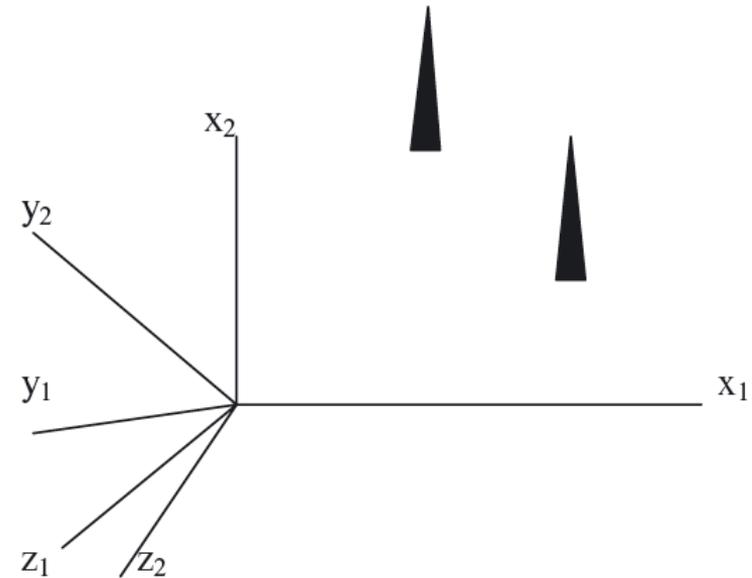
- ▶ Our examples involve 2 entangled particles.
- ▶ So our space must be at least $(3 \times N)D = 6D$.
 - ▶ 1st three coordinates = (x,y,z) coordinates for particle 1.
 - ▶ 2nd three coordinates = (x,y,z) coordinates for particle 2.
- ▶ Ψ_3 is (now) better represented by peaks at locations $(4,0,0,7,0,0)$ and $(7,0,0,4,0,0)$:



Distinctness of Ψ_3 and Ψ_4

$$\Psi_3 = \sqrt{\frac{1}{2}} |(4, 0, 0)\rangle_1 |(7, 0, 0)\rangle_2 \\ + \sqrt{\frac{1}{2}} |(7, 0, 0)\rangle_1 |(4, 0, 0)\rangle_2$$

$$\Psi_4 = \sqrt{1/2} |(4, 0, 0)\rangle_1 |(4, 0, 0)\rangle_2 \\ + \sqrt{1/2} |(7, 0, 0)\rangle_1 |(7, 0, 0)\rangle_2$$



Ney's primary claims (again)

- ▶ We've seen an argument for:
 - ▶ (A) “On any straightforward ontological reading of quantum mechanics, the theory requires the existence of an object, the wavefunction [...]
 - ▶ (B) The wavefunction inhabits an extremely high-dimensional space: configuration space. [...]
 - ▶ The wavefunction is a function from points in configuration space to probability amplitudes.
- ▶ What about (C)?
 - ▶ (C) No three of the many dimensions of configuration space correspond in any direct way with the three dimensions of our manifest image.”

Particles in configuration space

- ▶ The number of dimensions of configuration space **equals** the number of independent variables needed to specify the state of the world's wavefunction.
- ▶ In our examples we were specifying x,y,z coordinates for two entities (labelled 1 and 2) e.g.

$$\Psi_3 = \sqrt{\frac{1}{2}} |(4, 0, 0)\rangle_1 |(7, 0, 0)\rangle_2 + \sqrt{\frac{1}{2}} |(7, 0, 0)\rangle_1 |(4, 0, 0)\rangle_2$$

- ▶ That's why we needed $3N$ independent variables. This is not to say that fundamentally, there are two particles. Fundamentally there is just the wavefunction.

Particles in configuration space

- ▶ Fundamentally, there is the wavefunction.
- ▶ The question is whether we can view particles evolving in three dimensions as *derivative* on the wavefunction.
 - ▶ In the same way trees and tables are derivative on collections of particles in classical physics.
 - ▶ In the same way mental states are derivative on physical states according to physicalism.
 - ▶ The nature of this relation is disputed (“supervenience”, “grounding” etc.) but examples of it are easy to come up with.
- ▶ Is there a way of showing that particles evolving in three dimensions are nothing over and above the wavefunction?

Particles in configuration space

- ▶ According to Ney this is not easy.
 - ▶ Note how easy it is in relativity or string theory – ordinary three dimensions are identical with a subset of total dimensions.
- ▶ But: “No three of the dimensions of configuration space correspond to the three dimensions of our manifest image.” (p538).
- ▶ To illustrate this Ney asks us to consider three woodchips from her table...

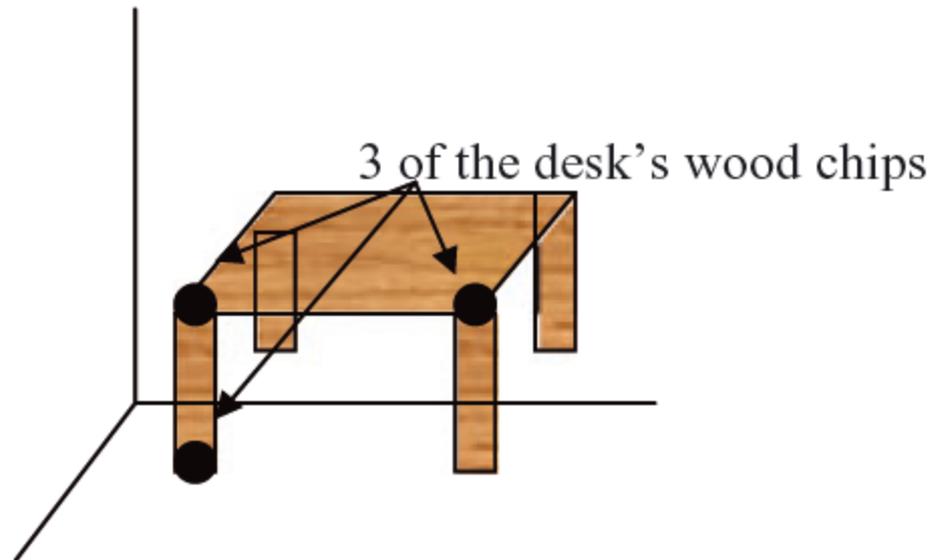
Finding 3D particles in 3N dimensions

- ▶ We start by assuming that 3 of the 3ND dimensions correspond to the x,y,z coordinates of one woodchip.

Chip 1: $x = 1$
 $y = 0$
 $z = 2$

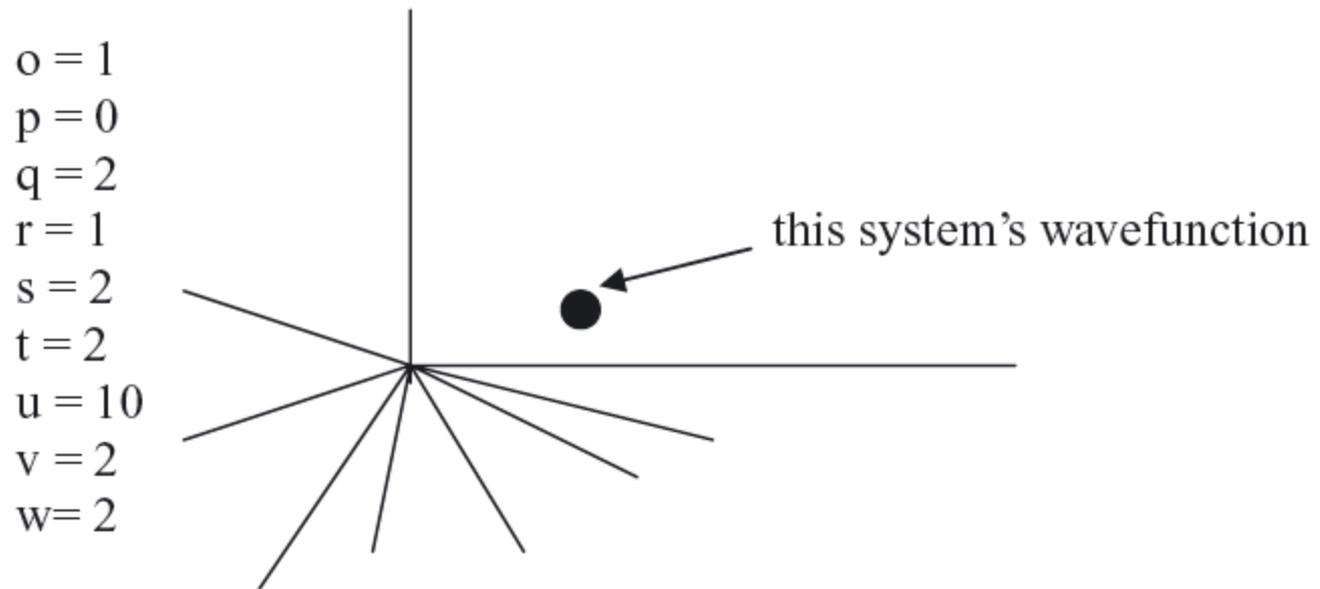
Chip 2: $x = 1$
 $y = 2$
 $z = 2$

Chip 3: $x = 10$
 $y = 2$
 $z = 2.$



Finding 3D particles in $3N$ dimensions

- ▶ We now consider the configuration space, with $3N$ dimensions ($N = \text{number of woodchips} = 3$) labelled o to w ...



- ▶ Which of the nine dimensions corresponds to the x dimension of our 3D space?

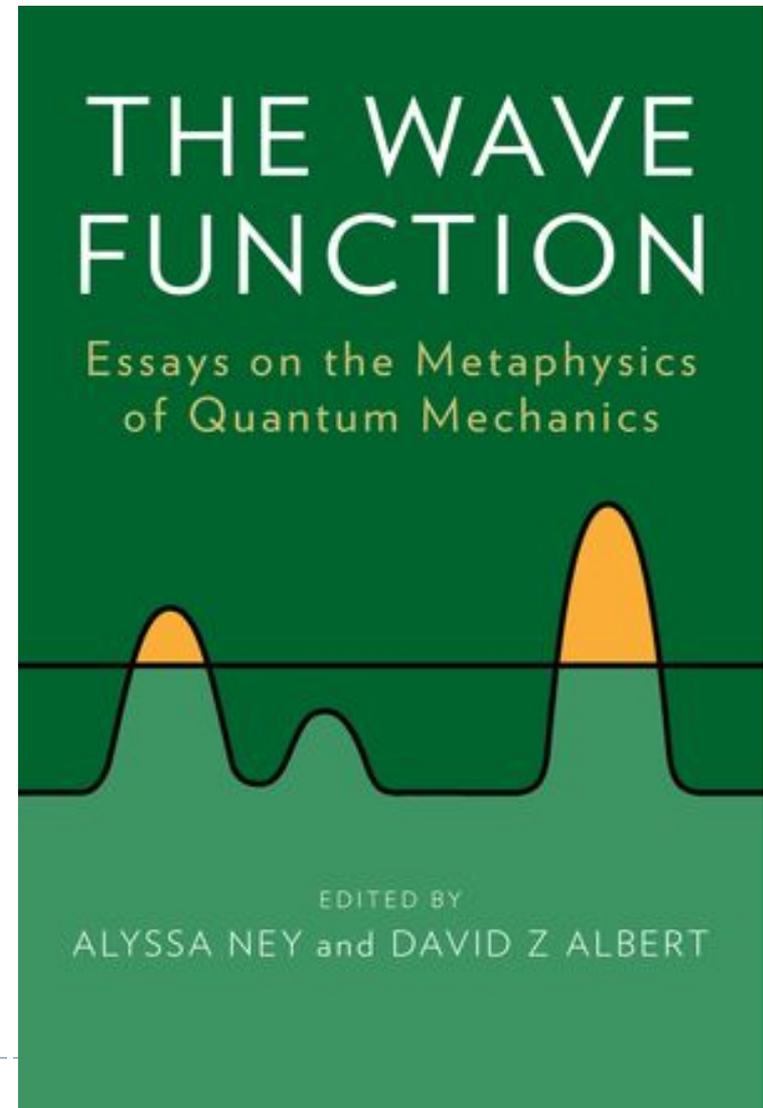
Finding 3D particles in $3N$ dimensions

- ▶ Which of the nine coordinates corresponds to the x-coordinate of our 3D space?
- ▶ None!
 - ▶ The o-coordinate corresponds to the x-coordinate for chip 1.
 - ▶ The r-coordinate corresponds to the x-coordinate for chip 2.
 - ▶ And so on.
- ▶ And no one of o, r, or u just is the x-dimension.

- ▶ Similarly we can ask: which coordinate in the 9D space corresponds to the height of the desk?
 - ▶ p, s, and f may appear to be relevant, but no one of them can be (e.g. p, at best, corresponds to the height of chip 1).
 - ▶ Something is supposed to be extended in the y-dimension. But nothing is extended in p (or in s or in f.)

Views on how to find 3D particles

- ▶ Ney and Albert recently edited a collection of essays in which various authors defend a variety of views.
 - ▶ “The Wave Function” 2013.
- ▶ Ney (2012: sections 4,5) evaluates many of these views.
- ▶ Let’s consider the main options...



Four views on how to find 3D particles

- ▶ **Two physical spaces, 3ND and 3D.**
 - ▶ The 3ND space of the wavefunction and the 3D space of ordinary matter.
- ▶ **One physical space, and it is 3ND.**
 - ▶ Three dimensional space is an illusion.
- ▶ **One fundamental physical space, and it is 3ND.**
 - ▶ Three dimensional space is derivative or emergent.
- ▶ **One physical space, and it is 3D.**
 - ▶ There is an error in Ney's (and other's) reasoning.

Two physical spaces, 3ND and 3D

- ▶ Some find this problem so troubling that they think it motivates *two* spaces!
- ▶ On this view one:
 - ▶ Concedes that the $3N$ dimensions of configuration space are physically real and that no one of them is (for e.g.) the x -dimension of three space.
 - ▶ But postulates an *additional* 3D space inhabited by ordinary matter, and laws connecting (for e.g.) the p , s , and f coordinates of 3ND space with the y -dimension of 3D space.

One physical space, and it is 3ND

- ▶ David Albert once suggested that our experiences of 3D space are illusions:
 - ▶ “[I]t has been essential to the project of quantum-mechanical realism [...] to learn to think of wave functions as physical objects *in and of themselves*. And of course the space those sorts of objects *live in*, and (therefore) the space we live in [...] is *configuration-space*. And whatever impression we have to the contrary (whatever impression we have, say, of living in a three-dimensional space, or in a four-dimensional spacetime) is somehow illusory.” (*Elementary Quantum Metaphysics* 1996).
- ▶ A return to Kant?
 - ▶ Psychophysical principles connecting (for e.g.) the p, s, and f coordinates of 3ND space with *an appearance of a y-dimension of 3D space*.

One fundamental physical space, 3ND

- ▶ Consider a 2D space with millions of tiny dots distributed on it, that compose a few large squares.
 - ▶ The dots are fundamental, but the squares are derivative or emergent.
 - ▶ Could 3D space be analogous to the squares, 3ND space analogous to the dots?
- ▶ Ney goes for a middle-ground between the last two views: there are ordinary macroscopic objects, only their three-dimensionality is illusory.
 - ▶ Somewhat underdeveloped though: can they retain enough properties that would justify the claim that they are ordinary macroscopic objects, if they're not 3D?

One physical space, and it is 3D

- ▶ Perhaps the reasoning in favour of 3ND is erroneous?
- ▶ Seems correct to say that none of the 3N dimensions is identical to any of the ordinary 3.
- ▶ But why did we postulate a 3ND space in the first place?
 - ▶ Because representing states Ψ_3 and Ψ_4 as instantiated in 3-space *required the same representation*, but the states are distinct.
 - ▶ Is this right?
 - ▶ Couldn't we add further information to the 3D representations to distinguish these states... entanglement properties?
 - ▶ But how would this work?